Lazy evaluation illustrated

for Haskell divers

exploring some mental models and implementations

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WIP

Lazy,... zzz

..., It's fun!

NOTE

- Meaning of terms are different for each community.
- There are a lot of good documents. Please see also references.
- This is written for GHC's Haskell.

Contents

1. Introduction

1. Introduction

Basic mental models

How to evaluate a program in your brain ?

a program

code
code
code
:

?

How to evaluate (execute, reduce) the program in your brain?

What "mental model" do you have?

One of the mental models for C program

C program

A sequence of statements

```
main (...) {
  code..
  code..
  code..
  code..
}
```

?

A nested structure

x = func1(func2(a));

?

A sequence of arguments

y = func1(a(x), b(x), c(x));

?

A function and arguments

z = func1(m + n);

?

How to evaluate (execute, reduce) the program in your brain?

What step, what order, ... ?

One of the mental models for C program

C program

A program is a collection of statements.

A sequence of statements

```
main (...) {
  code..
  code..
  code..
  code..
}
```

Statements are executed downward.

A nested structure

x = func1(func2(a));

from inner to outer

A sequence of arguments

y = func1(a(x), b(x), c(x));

from left to right

A function and arguments

z = func1(m + n);

arguments first
apply second

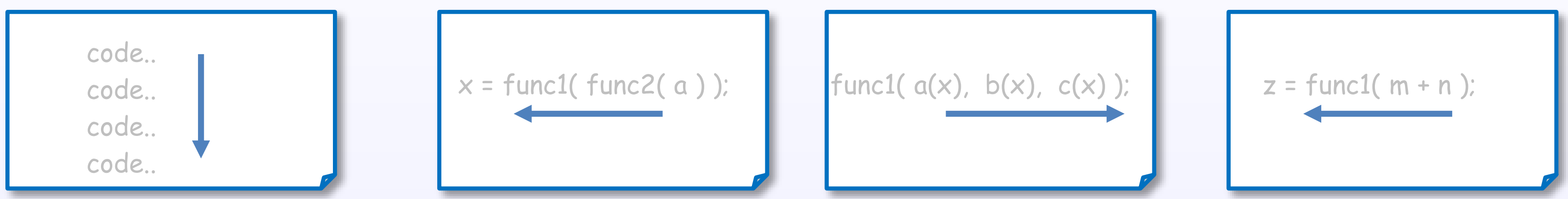
Each programmer has some mental models in their brain.

One of the mental models for C program

Maybe, You have some implicit mental model in your brain for C program.

(1) A program is a collection of statements.

(2) There is the order between evaluations of elements.



(3) There is the order between termination and start of evaluations.

This is a syntactically straightforward model for programming languages.

(an implicit sequential order model)

One of the mental models for Haskell program

Haskell program

$$
\left.\begin{aligned}
main &= exp_{aa}\ (exp_{ab}\ exp_{ac}\ exp_{ad})\\
exp_{ac} &= exp_{aca}\ exp_{acb}\\
exp_{ad} &= exp_{ada}\ exp_{adb}\ exp_{adc}\\
&\ \vdots
\end{aligned}\right\}\ ?
$$

How to evaluate (execute, reduce) the program in your brain?

What step, what order, ... ?

One of the mental models for Haskell program

Haskell program

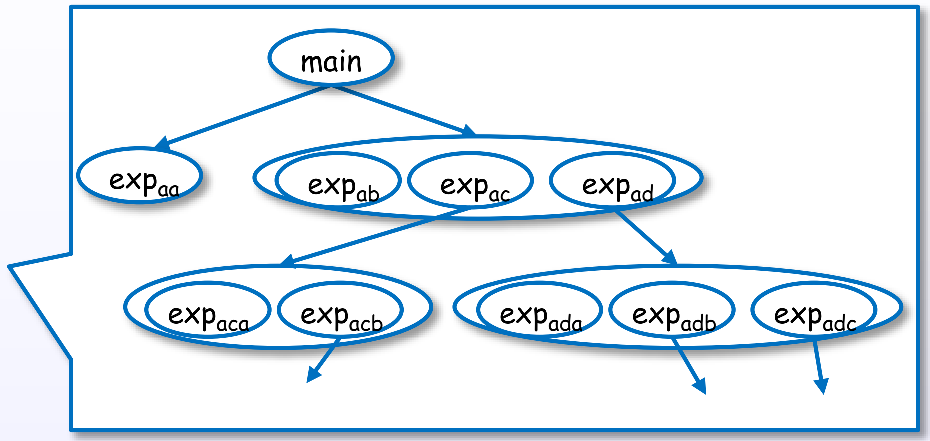
A program is a collection of expressions.

main = $\underline{exp_{aa}}$ $\underline{(exp_{ab}\ \ exp_{ac}\ exp_{ad}\ )}$

$\underline{exp_{ac}}$ = exp_{aca} exp_{acb}

$\underline{exp_{ad}}$ = exp_{ada} exp_{adb} exp_{adc}

:



main = exp_{aa} $(exp_{ab}\ (exp_{aca}\ exp_{acb}\ )\ (exp_{ada}\ exp_{adb}\ exp_{adc})\ )$

A entire program is regarded as a single expression.

The subexpression is evaluated (reduced) in some order.

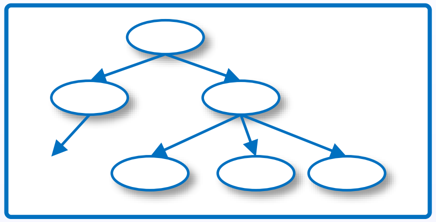
The evaluation is performed by replacement.

One of the mental models for Haskell program

(1) A program is a collection of expressions.

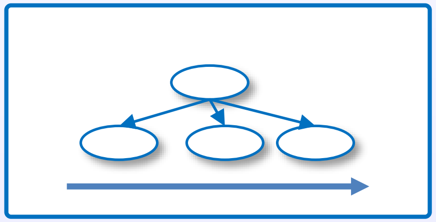
(2) A entire program is regarded as a single expression.

```
main = e (e (e (e e) e (e e e) ) )
```

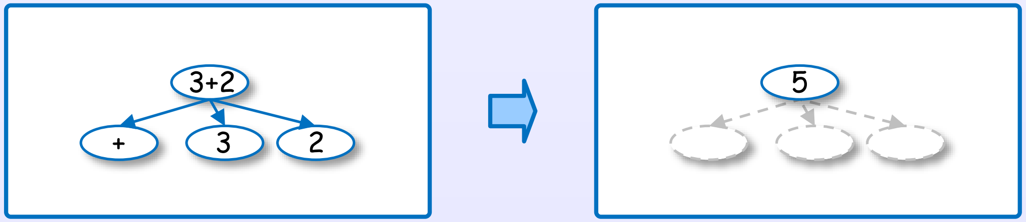


(3) The subexpressions are evaluated (reduced) in some order.

```
f = e (e (e (e e) e (e e e) ) )
```



(4) The evaluation is performed by replacement.



This is an example of an expression reduction model for Haskell.

Lazy evaluation

Why lazy evaluation?

To manipulate infinite data structures

To avoid unnecessary computation

To manipulate streames

modularity

pure is order free

abstraction

amortizing

To manipulate huge data structures

potentially parallelism

2nd Church-Rosser theorem

out-of-order optimization

To implement non-strict semantics

fun

asynchronization

reactive

...

There are various reasons ☺

References : [H4], [H3], [B2], [B7], [B8], [D2], [D12], [D13], [D14]

Haskell(GHC) 's lazy evaluation

Lazy evaluation

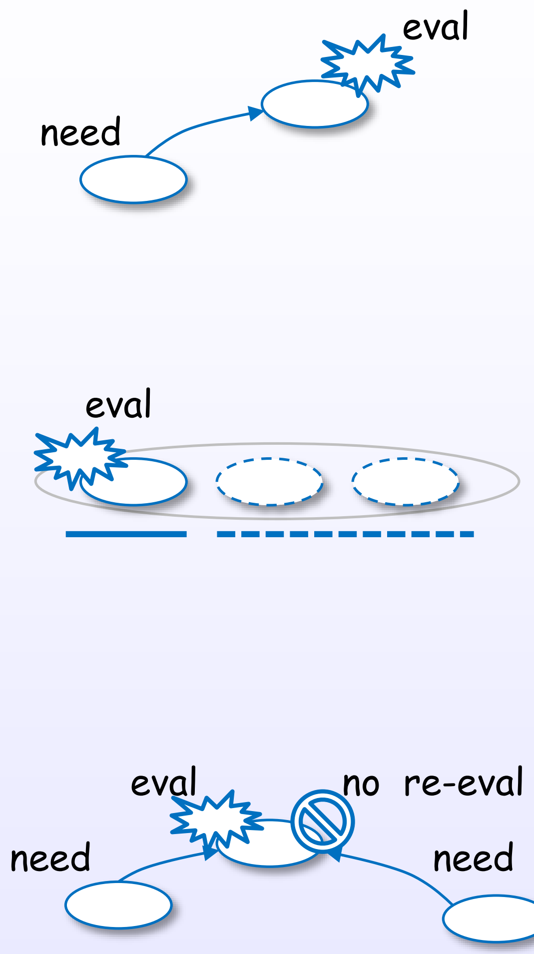
evaluate only when needed

+

evaluate only enough

+

evaluate at most once

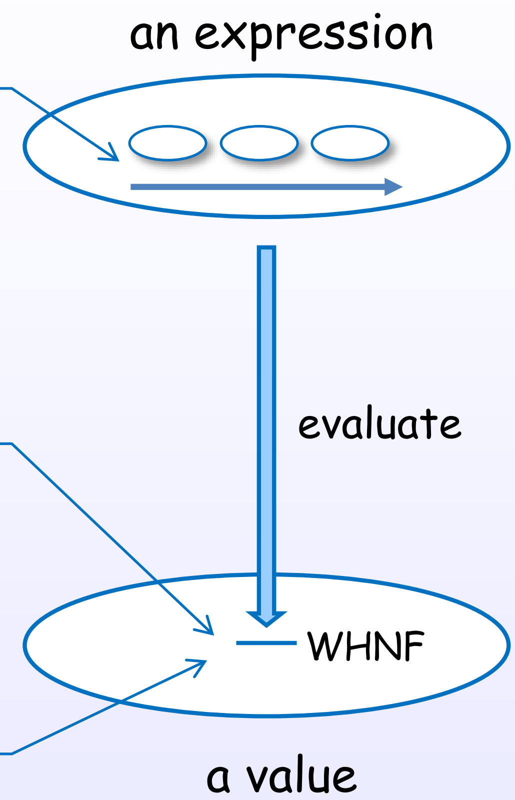


"Lazy" is "delay and avoidance" rather than "delay".

References : [B2] Ch.7, [H4] Ch.11, 12, [D2]

Ingredient of Haskell(GHC) 's lazy evaluation

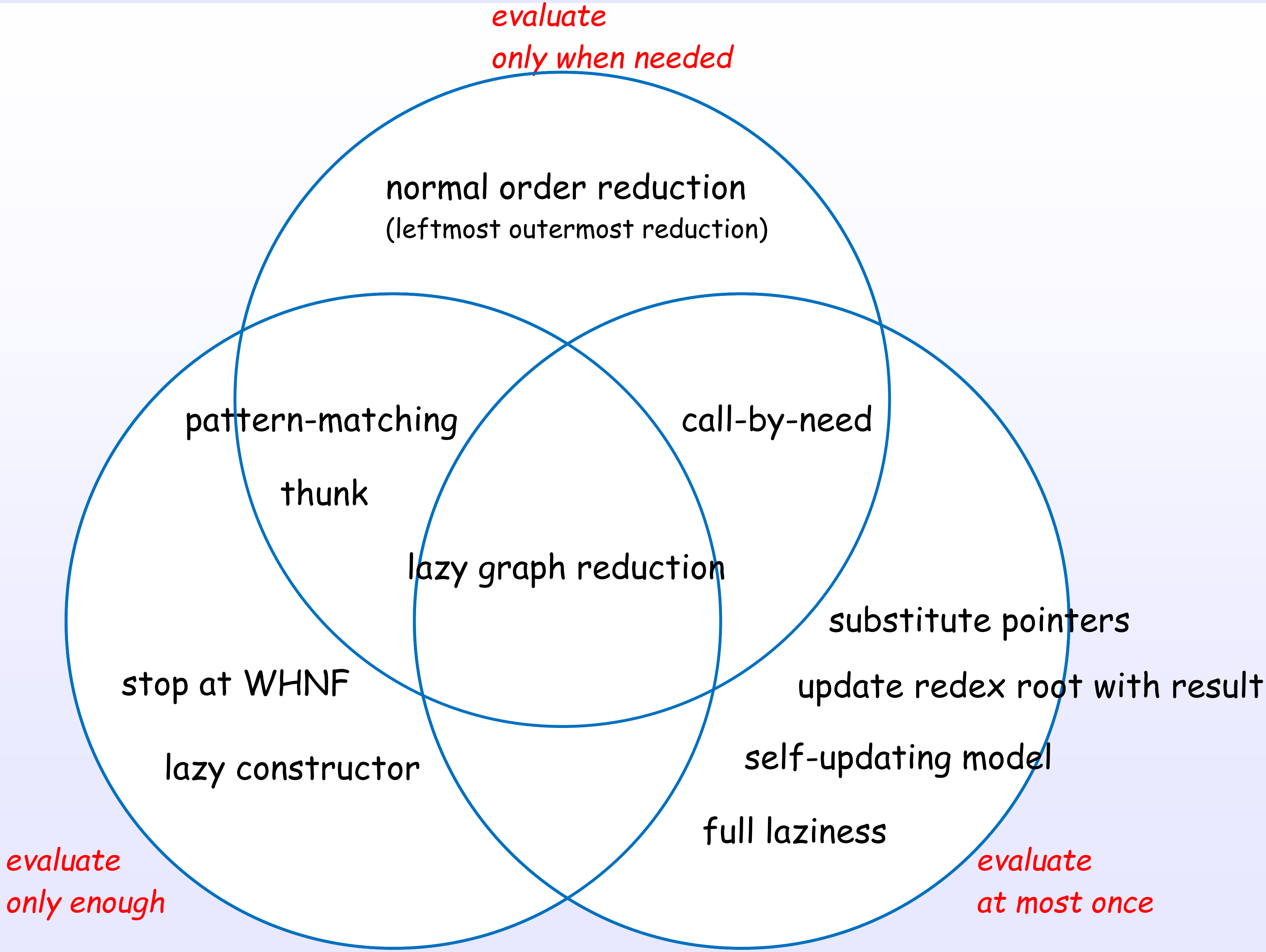
only when needed	normal order reduction
+	
only enough	stop at WHNF
+	
at most once	substitute pointers update redex root with result



This strategy is implemented by lazy graph reduction.

References : [B2] Ch.7, [H4] Ch.11, 12, [D2]

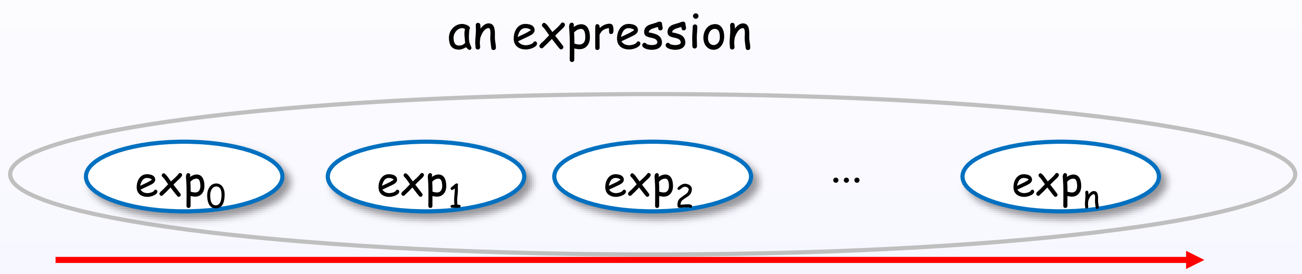
Techniques of Haskell(GHC) 's lazy evaluation



References : [B2] Ch.7, [H4] Ch.2, 11, 12, 15, [H5], [D2]

Simple questions

What order?



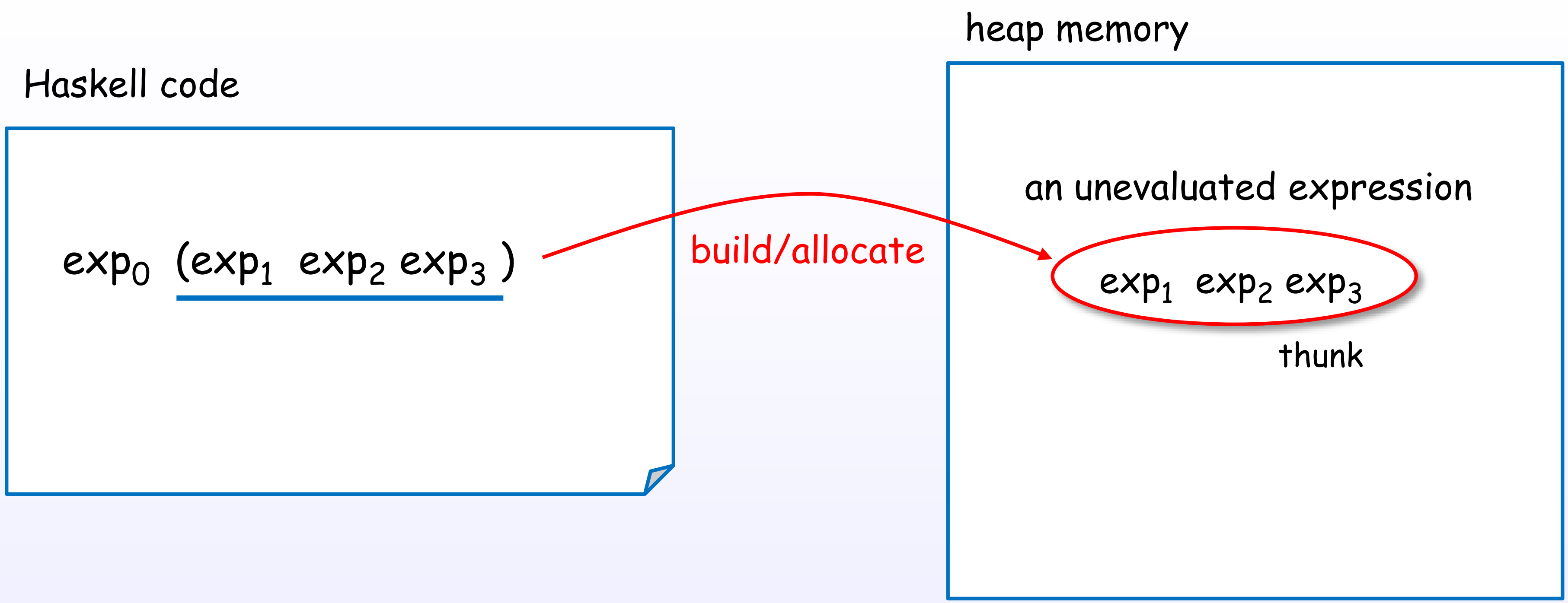
An expression is evaluated by normal order (leftmost outermost redex first).

Normal order reduction guarantees to find a normal form (if one exists).

To avoid unnecessary computation, normal order reduction chooses to apply the function rather than first evaluating the argument.

References : [H4] Ch.2, 11, [B6] Ch.5

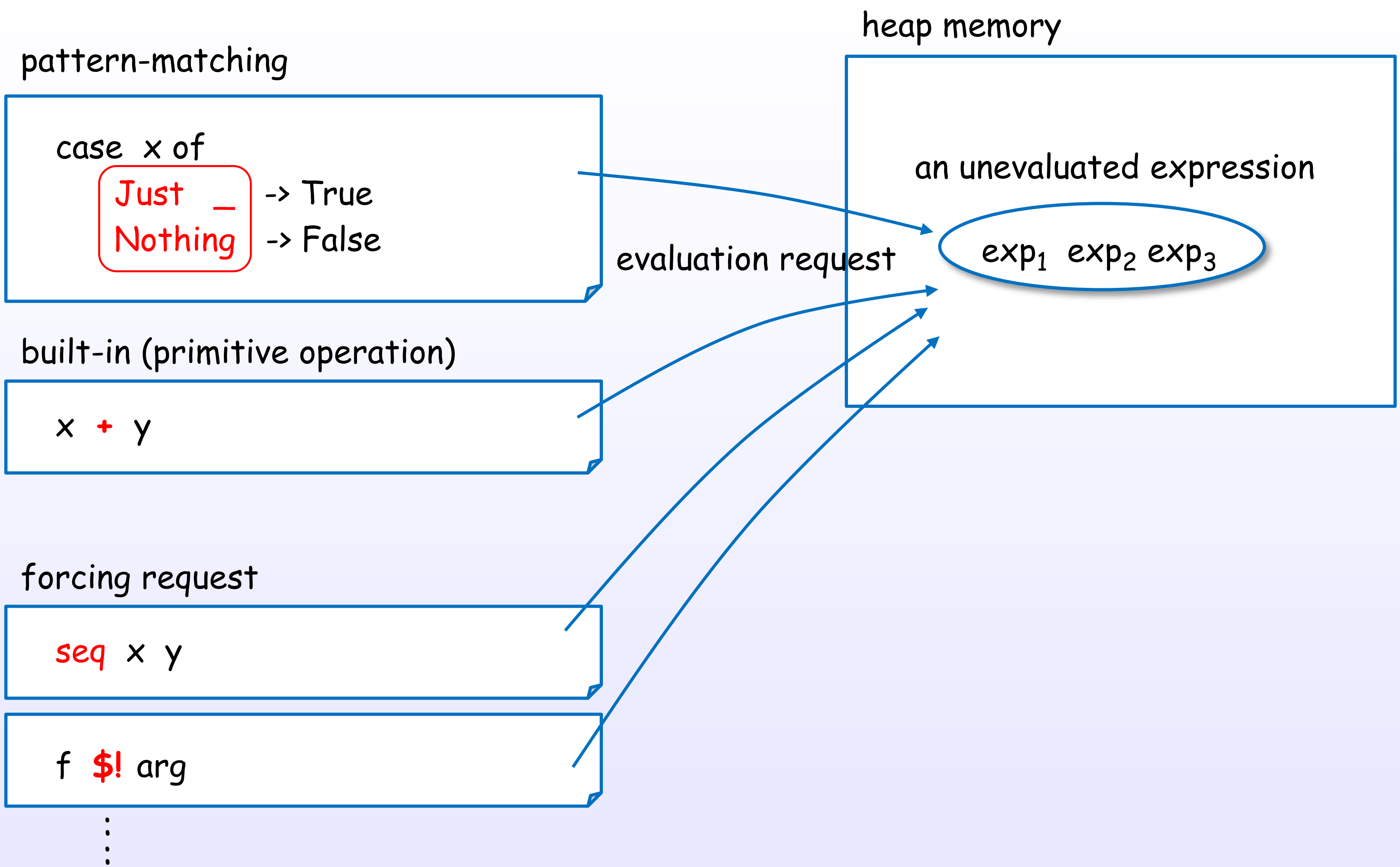
How to postpone?



To postpone the evaluation, an unevaluated expression is built in the heap memory.

References : [H4], [H5]

When needed?

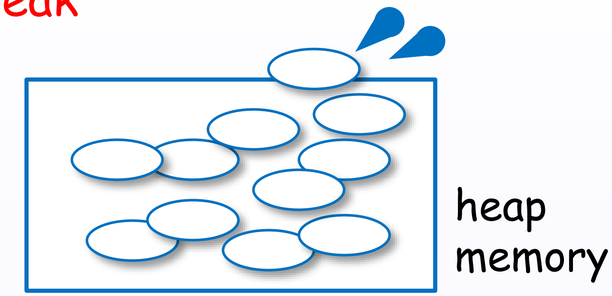


Pattern-matching or forcing request drive the evaluation.

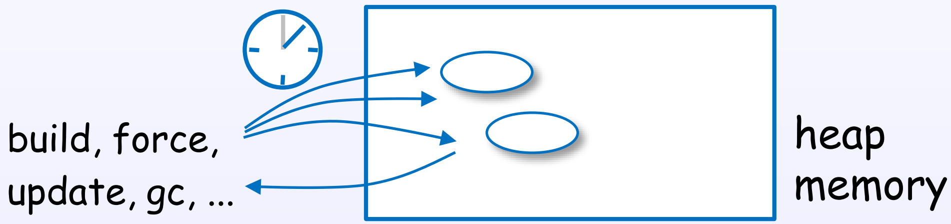
References : [H4], [D2], [D5]

What to be careful about?

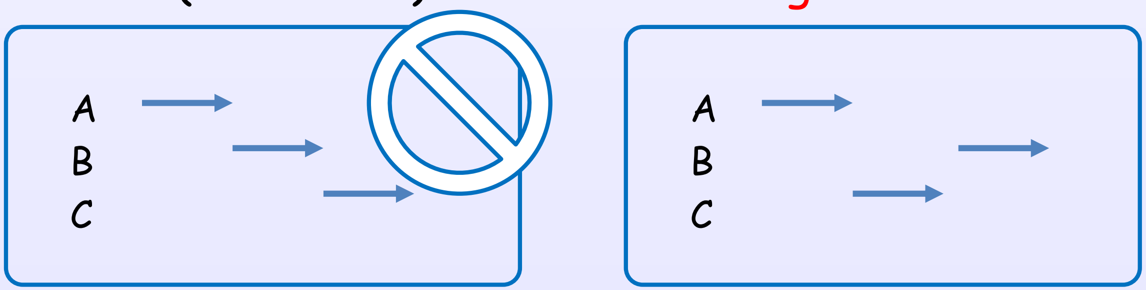
To consider hidden space leak



To consider performance cost to postpone unevaluated expressions



To consider evaluation (execution) order and timing in real world



You can avoid the pitfalls by controlling the evaluation.

References : [H4], [D2], [D5]

2. Expressions

Expression and value

What is an expression?

An expression

?

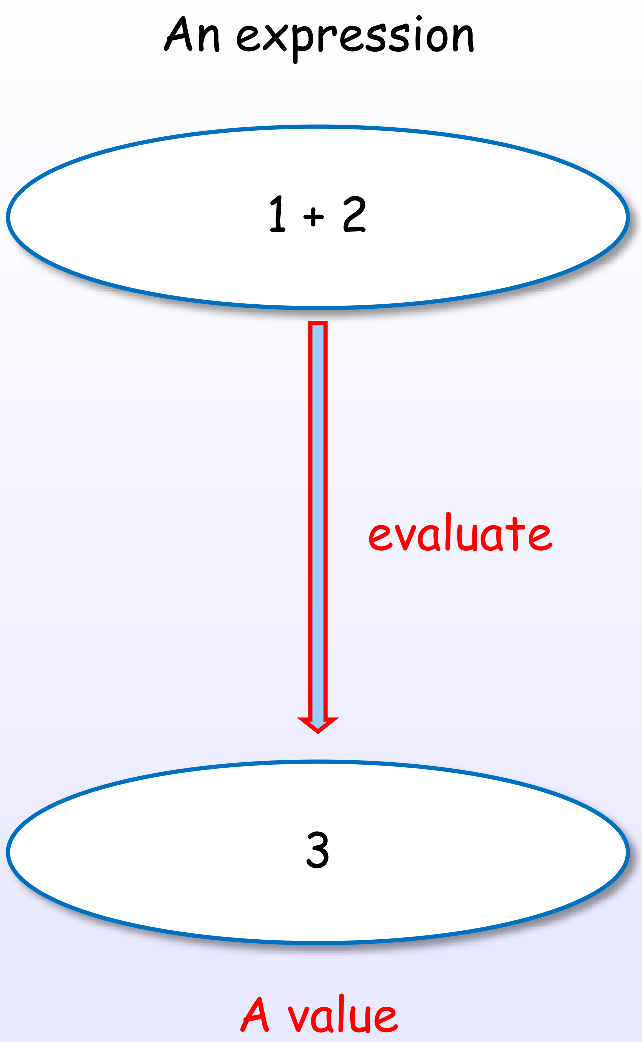
An expression denotes a value

An expression

1 + 2

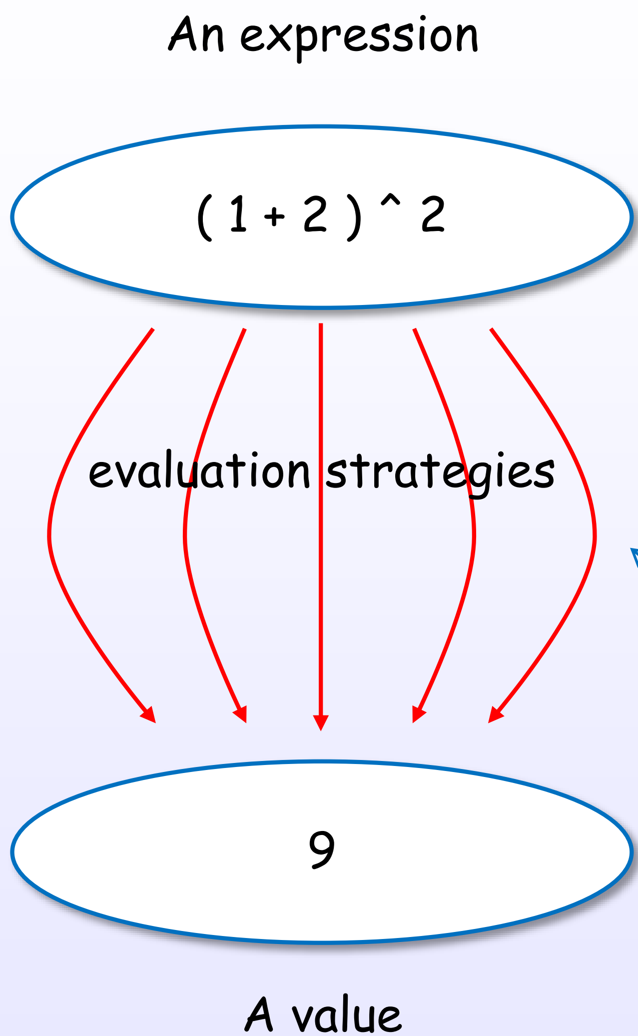
References : [B1] Ch.1, [B2] Ch.2, [H1] Ch.1, [B6] Ch.3, [H4] Ch.2

An expression is evaluated to a value



References : [B1] Ch.1, [B2] Ch.2, [H1] Ch.1, [B6] Ch.3, [H4] Ch.2

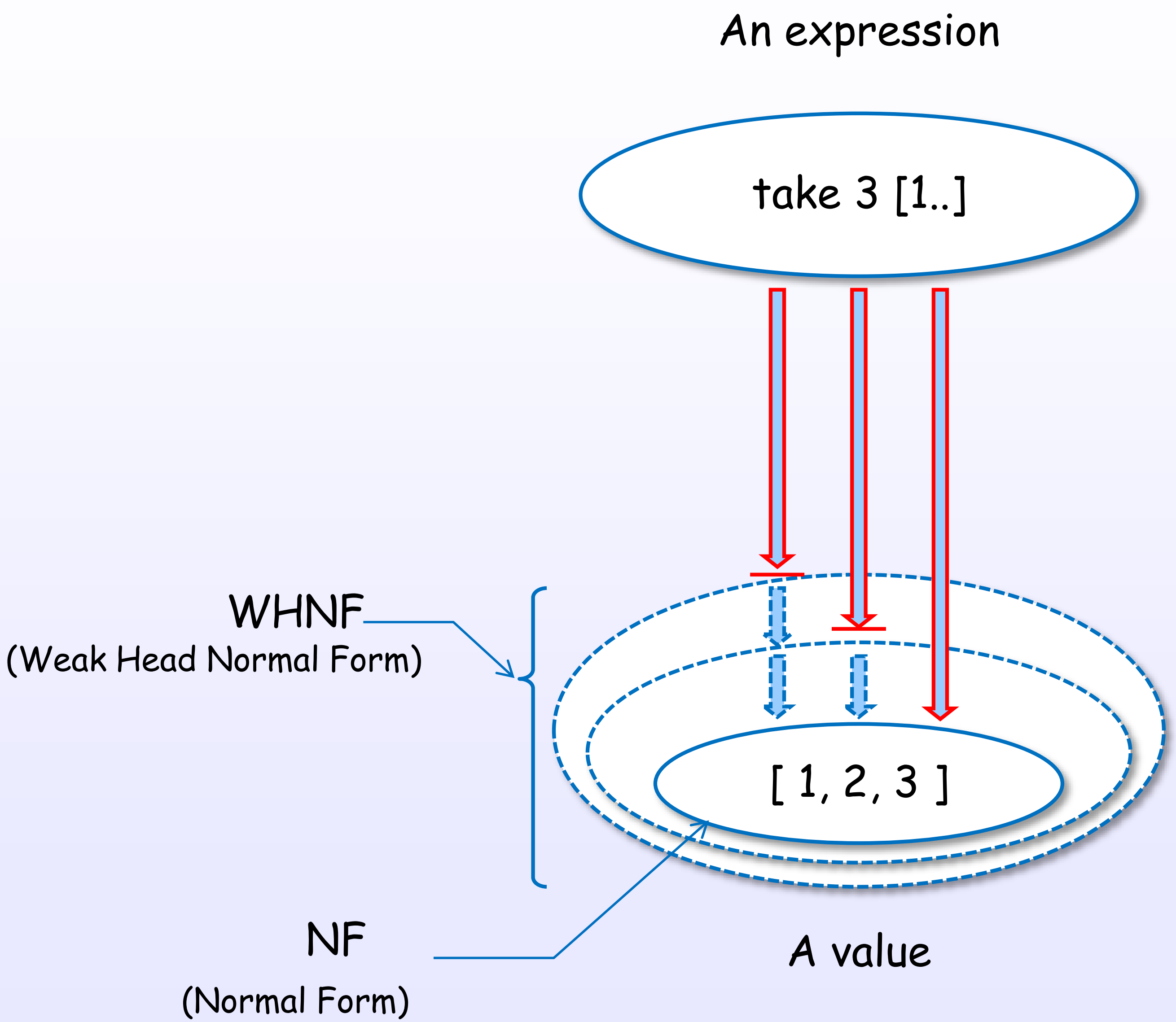
There are many evaluation approaches



- Strict, Non-strict evaluation
- Eager, Lazy evaluation
- Call-by-value, Call-by-name, Call-by-need, …
- Innermost, Outermost
- Normal order, Applicative order
- …

References : [B2] Ch.2, 7, [B6] Ch.3, [D1]

There are some evaluation levels



References : [D3], [B2] Ch.2, 7, [B6] Ch.3, [D1]

Expressions in Haskell

There are many expressions in Haskell

Expressions

- Just 5
- 1 + 2
- (1, 2)
- take 5 xs
- [1, 2, 3]
- let x = 1 in x + y
- 'a'
- map f xs
- if b then 1 else 0
- 7
- \ x -> x + 1
- x : xs
- fun arg
- case x of _ -> 0
- (\ x -> x + 1) 3
- do {x <- get; put x}
- xs

↓ categorizing

References : [B2] Ch.2, [H1] Ch.3

Expression categories in Haskell

lambda abstraction

```
\ x -> x + 1
```

let expression

```
let x = 1 in x + y
```

conditional

```
if b then 1 else 0
```

case expression

```
case x of _ -> 0
```

do expression

```
do {x <- get; put x}
```

function application

```
take 5 xs
( \ x -> x + 1) 3
1 + 2
map f xs
fun  arg
```

general constructor, literal and some forms

```
7
[1, 2, 3]
(1, 2)
'a'
x : xs
Just 5
```

variable

```
xs
```

References : [H1] Ch.3, [B2] Ch.2

Specification is described in Haskell 2010 Language Report

"Haskell 2010 Language Report, Chapter 3 Expressions" [H1]

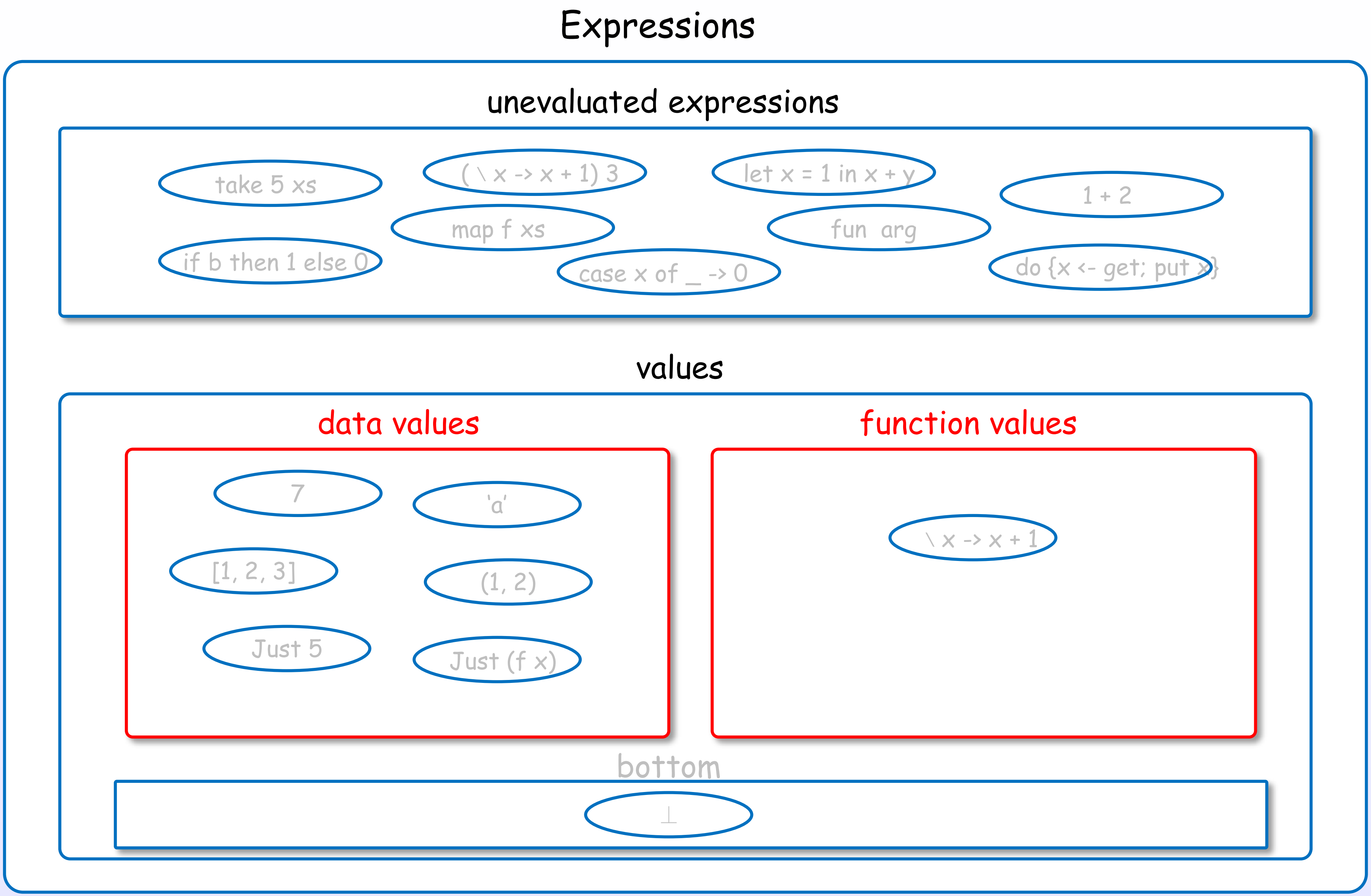
exp	→	*infixexp* `::` [*context* `=>`] *type*	(expression type signature)
	|	*infixexp*	
infixexp	→	*lexp qop infixexp*	(infix operator application)
	|	`-` *infixexp*	(prefix negation)
	|	*lexp*	
lexp	→	`\` $apat_1$ … $apat_n$ `->` *exp*	(lambda abstraction, $n \geq 1$)
	|	`let` *decls* `in` *exp*	(let expression)
	|	`if` *exp* [`;`] `then` *exp* [`;`] `else` *exp*	(conditional)
	|	`case` *exp* `of` `{` *alts* `}`	(case expression)
	|	`do` `{` *stmts* `}`	(do expression)
	|	*fexp*	
fexp	→	[*fexp*] *aexp*	(function application)
aexp	→	*qvar*	(variable)
	|	*gcon*	(general constructor)
	|	*literal*	
	|	`(` *exp* `)`	(parenthesized expression)
	|	`(` exp_1 `,` … `,` exp_k `)`	(tuple, $k \geq 2$)
	|	`[` exp_1 `,` … `,` exp_k `]`	(list, $k \geq 1$)
	|	`[` exp_1 [`,` exp_2] `..` [exp_3] `]`	(arithmetic sequence)
	|	`[` *exp* `|` $qual_1$ `,` … `,` $qual_n$ `]`	(list comprehension, $n \geq 1$)
	|	`(` *infixexp qop* `)`	(left section)
	|	`(` $qop_{\langle - \rangle}$ *infixexp* `)`	(right section)
	|	*qcon* `{` $fbind_1$ `,` … `,` $fbind_n$ `}`	(labeled construction, $n \geq 0$)
	|	$aexp_{\langle qcon \rangle}$ `{` $fbind_1$ `,` … `,` $fbind_n$ `}`	(labeled update, $n \geq 1$)

References : [H1] Ch.3, [B2] Ch.2

2. Expressions

Classification by values and forms

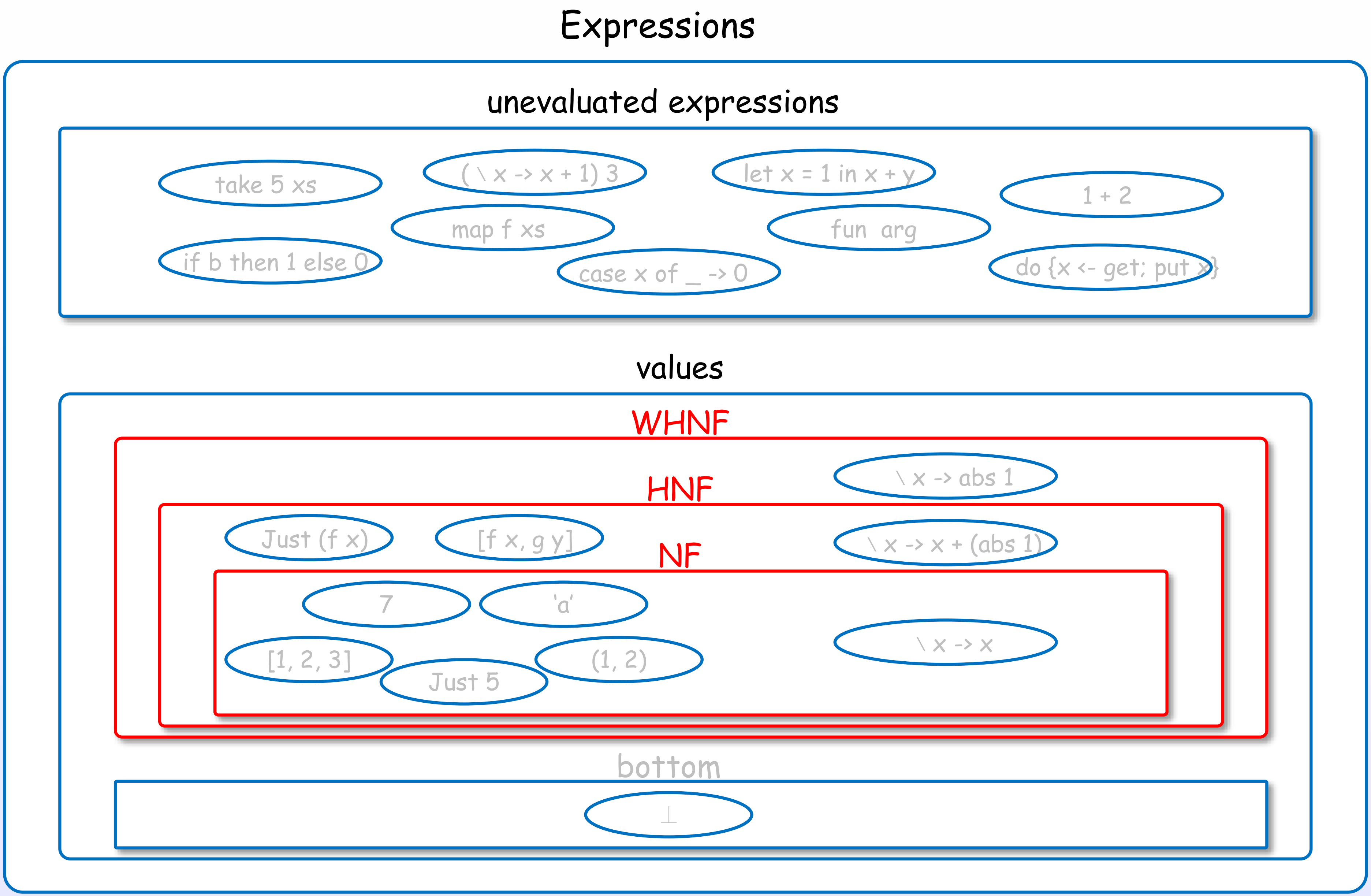
Classification by values



Values are data values or function values.

References : [H5]

Classification by forms

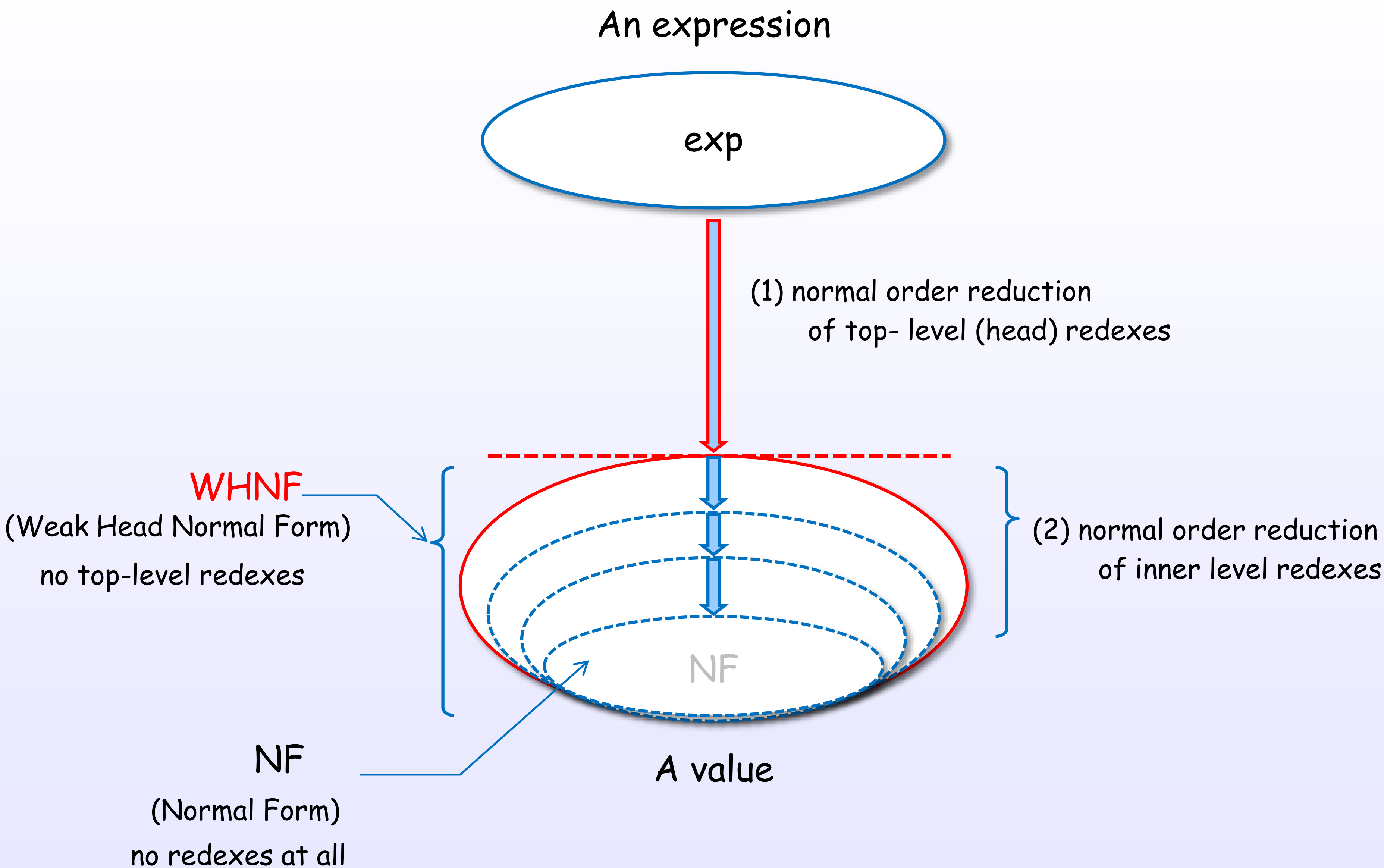


Values are WHNF, HNF or NF.

References : [H4] Ch.11, [D3], [B6] Ch.3, [B2] Ch.2, 7, [D1], [W1]

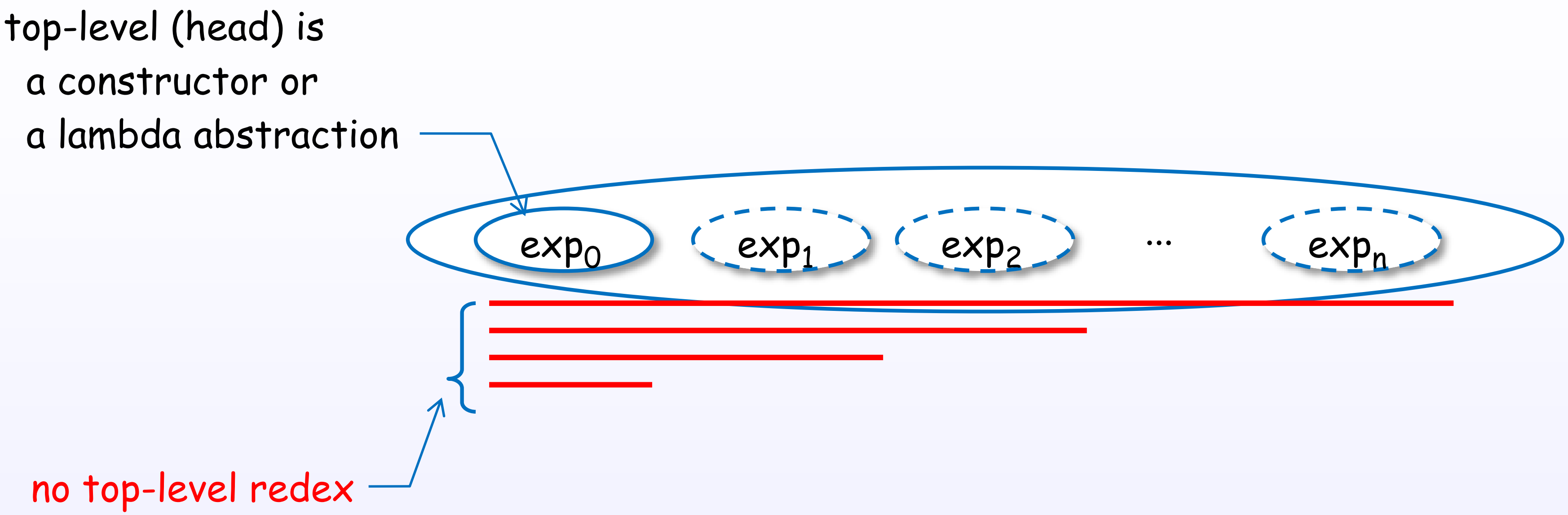
WHNF

WHNF is one of the form in the evaluated values



References : [H4] Ch.11, [D3], [B6] Ch.3, [B2] Ch.2, 7, [D1], [W1]

WHNF

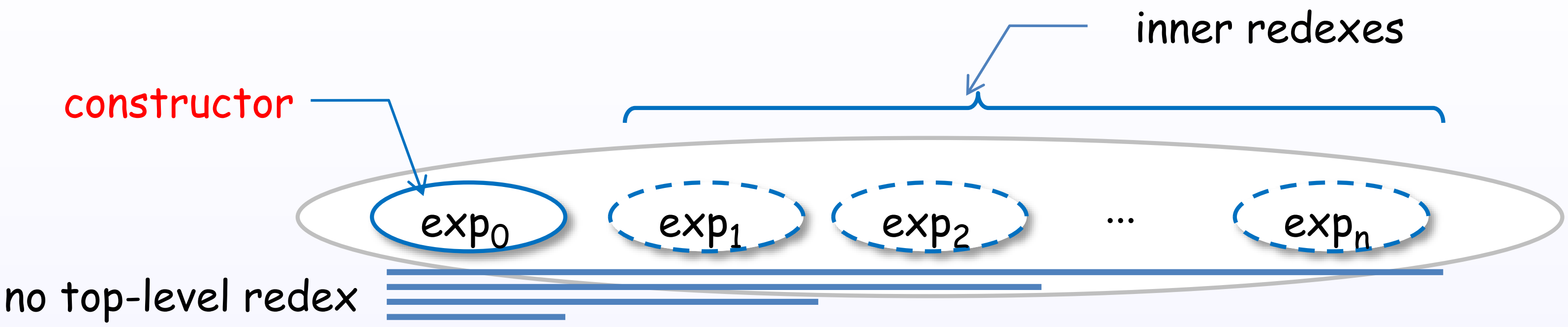


WHNF is a value which has evaluated top-level

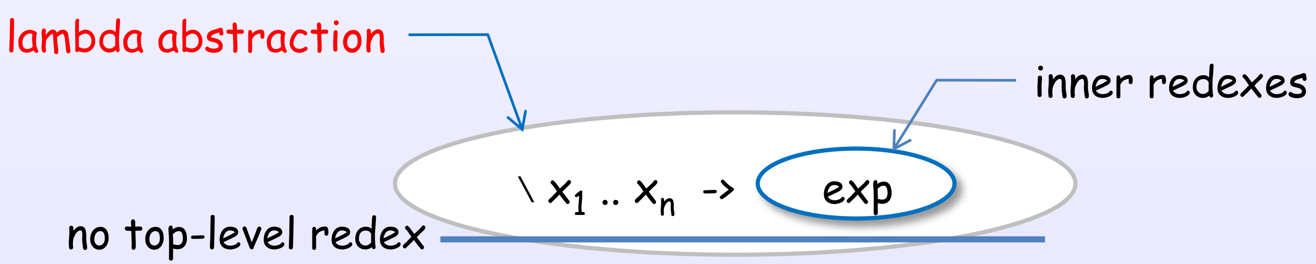
References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

WHNF for a data value and a function value

a data value in WHNF



a function value in WHNF



References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

Examples of WHNF

WHNF

Just 7

no top-level redex

Just (abs x)

no top-level redex

Cons (f 1) (map f [2..])

no top-level redex

\ x -> x + 1

no top-level redex

---

no WHNF

abs 7

top level-redex

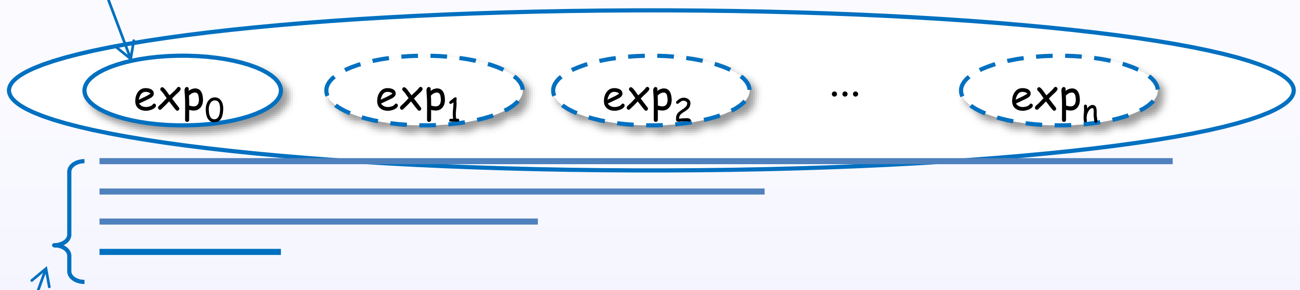
if x then True else False

top level-redex

References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

HNF

top-level (head) is
a constructor or
a lambda abstraction with no top-level redex



no top-level redex

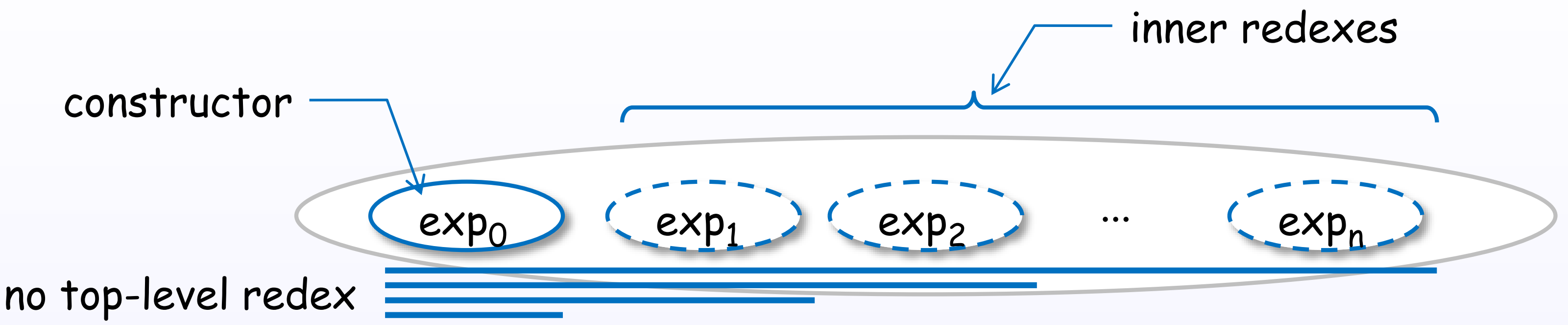
HNF is a value which has evaluated top-level

* GHC uses WHNF rather than HNF.

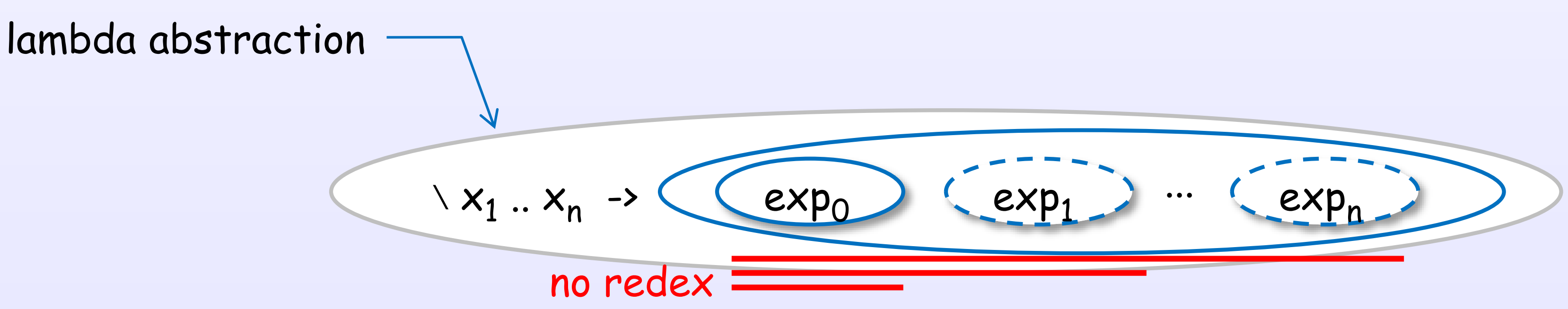
References : [H4] Ch.11, [D3], [B3]

HNF for a data value and a function value

a data value in HNF (same as WHNF)



a function value in HNF



References : [H4] Ch.11, [D3], [B3]

Examples of HNF

HNF

Just 7

no top-level redex

Just (abs x)

no top-level redex

\ x -> Just (abs 7)

no top-level redex

---

no HNF

abs 7

top level-redex

\ x -> abs 7

top level-redex

References : [H4] Ch.11, [D3], [B3]

NF

top-level (head) is
a constructor or
a lambda abstraction

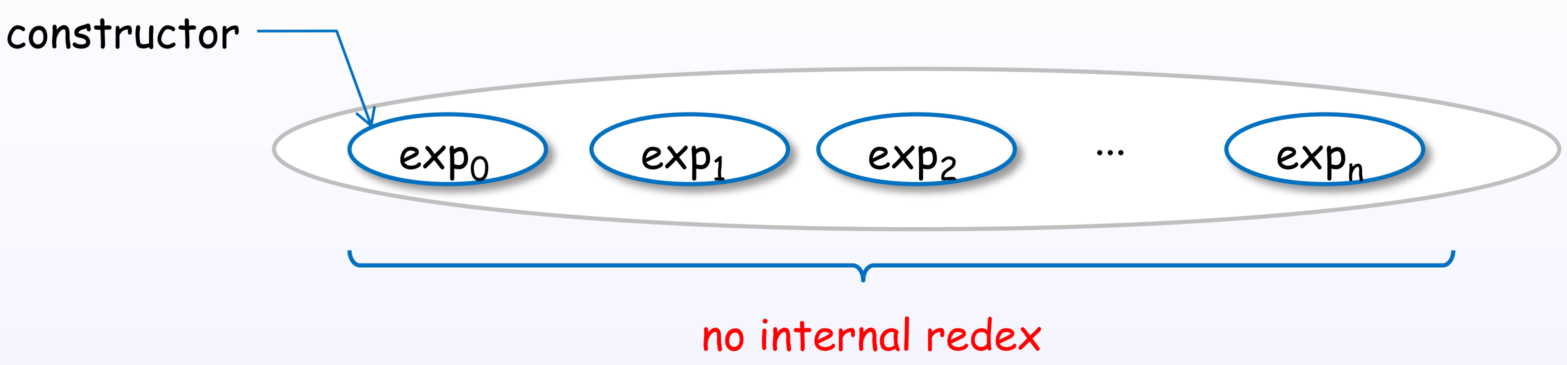
$$\underbrace{exp_0 \quad exp_1 \quad exp_2 \quad \cdots \quad exp_n}_{\text{no internal redex}}$$

NF is a value which has no redex.

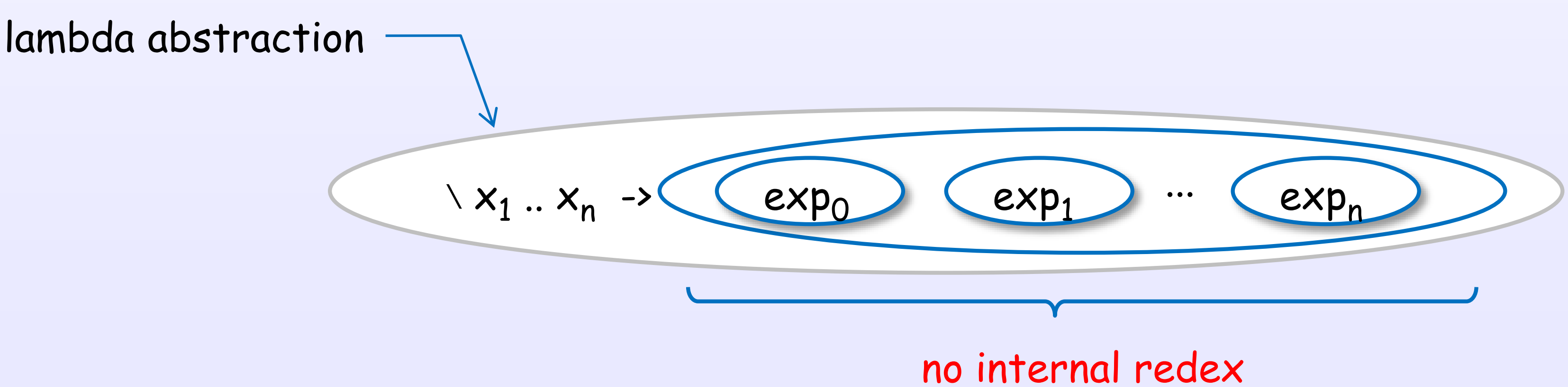
References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

NF for a data value and a function value

a data value in NF



a function value in NF



References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

Examples of NF

NF

$\underbrace{\texttt{Just 7}}_{\text{no internal redex}}$

$\underbrace{\texttt{Cons 1 Nil}}_{\text{no internal redex}}$

$\underbrace{\texttt{\textbackslash x -> x + 1}}_{\text{no internal redex}}$

---

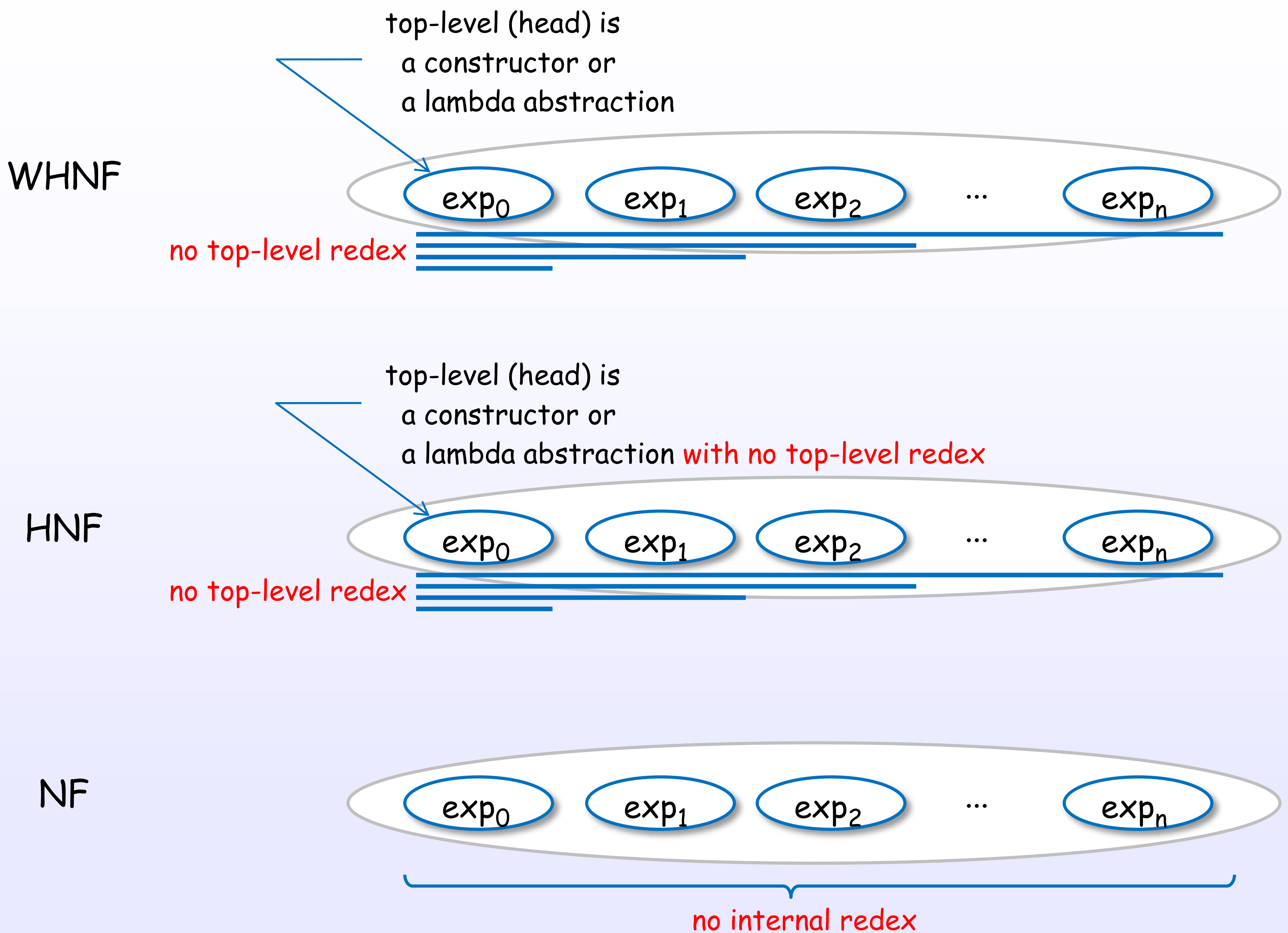
no NF

Just $\underbrace{\texttt{(abs 7)}}_{\text{redex}}$

\ x -> Just $\underbrace{\texttt{(abs 7)}}_{\text{redex}}$

References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

WHNF, HNF, NF



References : [H4] Ch.11, [D3], [B5] Ch.2, [B4] Ch.24, [B6] Ch.3, [B2] Ch.2, 7, [D5], [D1]

Definition of WHNF and HNF

"The implementation of functional programming languages" [H4]

11.3.1 Weak Head Normal Form

To express this idea precisely we need to introduce a new definition:

DEFINITION

A lambda expression is in *weak head normal form* (WHNF) if and only if it is of the form

$$F\ E_1\ E_2\ \ldots\ E_n$$

where $n \geq 0$;
and either F is a variable or data object
or F is a lambda abstraction or built-in function
and $(F\ E_1\ E_2\ \ldots\ E_m)$ is not a redex for any $m \leq n$.

An expression has no *top-level redex* if and only if it is in weak head normal form.

11.3.3 Head Normal Form

Head normal form is often confused
some discussion. The content of th
since for most purposes head norma
form. Nevertheless, we will stick to t

DEFINITION

A lambda expression is in *head normal form* (HNF) if and only if it is of the form

$$\lambda x_1.\lambda x_2 \ldots \lambda x_n.(v\ M_1\ M_2\ \ldots\ M_m)$$

where $n, m \geq 0$;
v is a variable (x_i), a data object, or a built-in function;
and $(v\ M_1\ M_2\ \ldots\ M_p)$ is not a redex for any $p \leq m$.

References : [H4] Ch.11

3. Internal representation of expressions

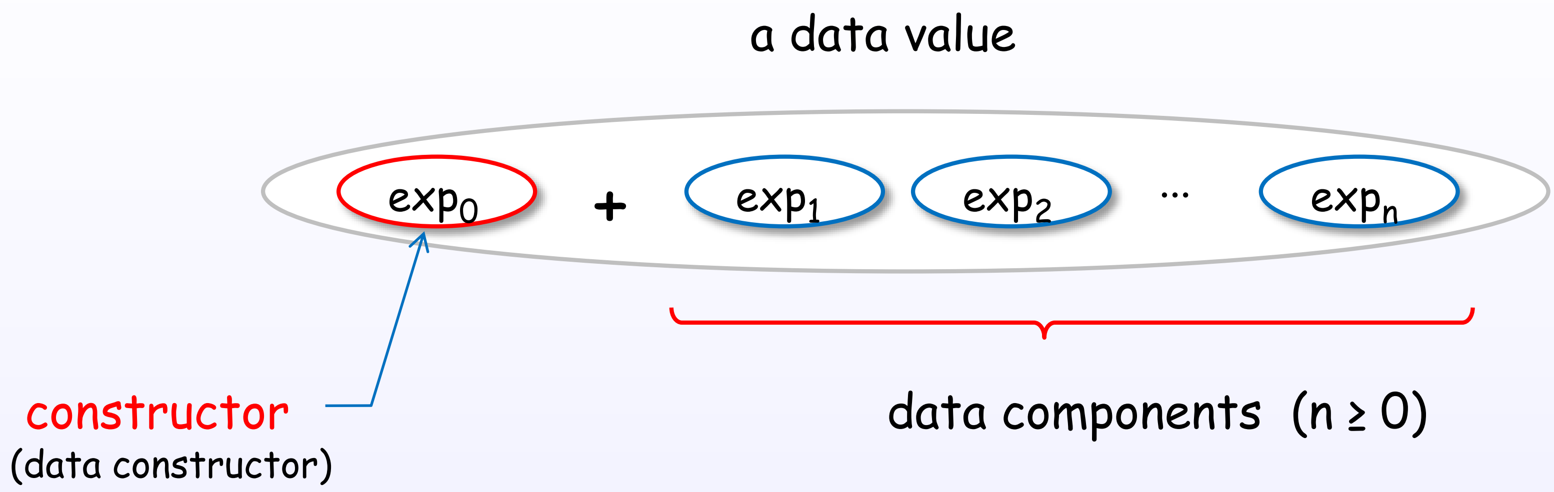
3. Internal representation of expressions

Constructor

Constructor

Constructor is one of the key elements
to understand WHNF and lazy evaluation in Haskell.

Constructor



A constructor builds a structured data value.

A constructor distinguishes the data value in expressions.

References : [B2], [H1], [H4] Ch.2, 10, [B6] Ch.11

Constructors and data declaration

constructor

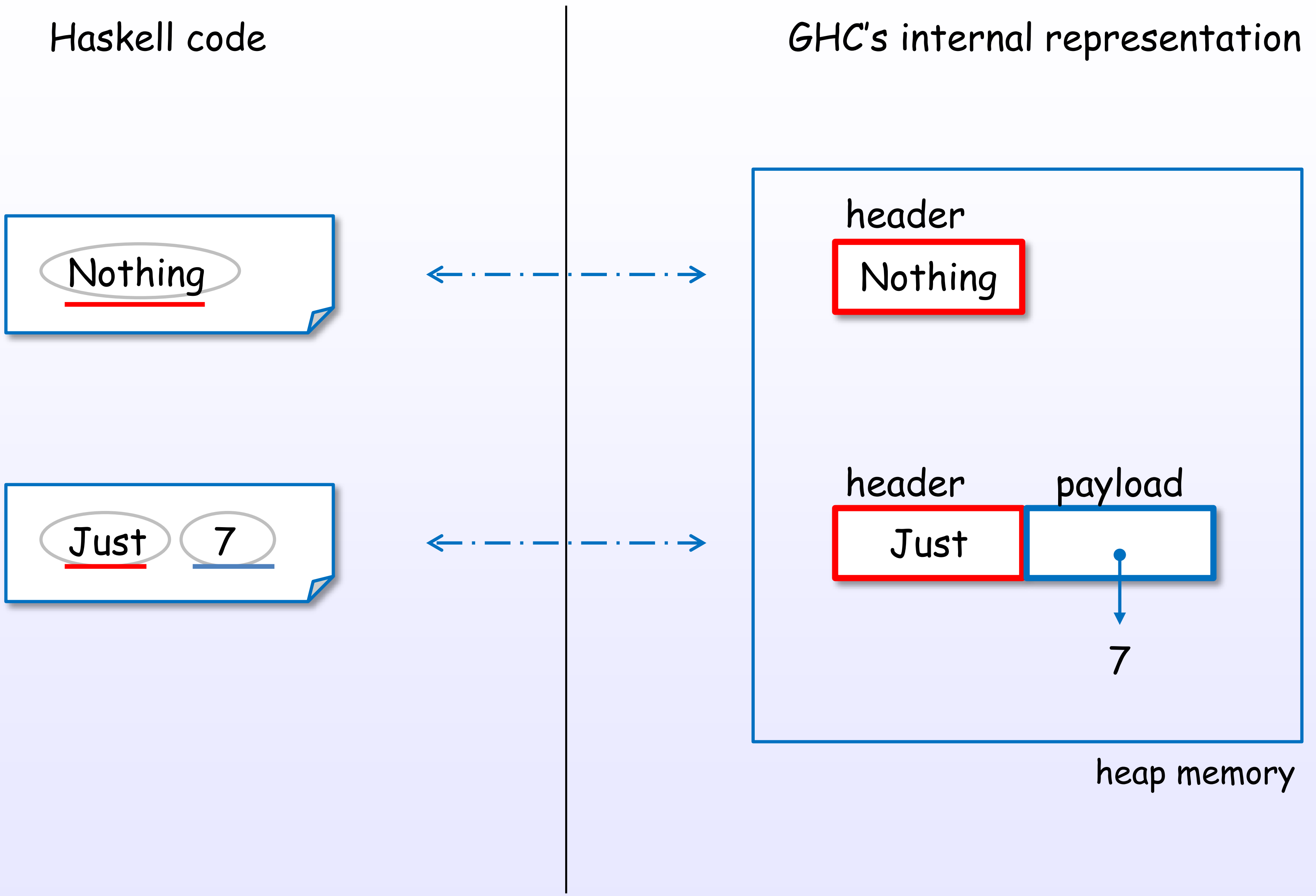
data component

```
data  Maybe a =  Nothing
              |  Just  a
```

Constructors are defined by data declaration.

References : [B2], [H1]

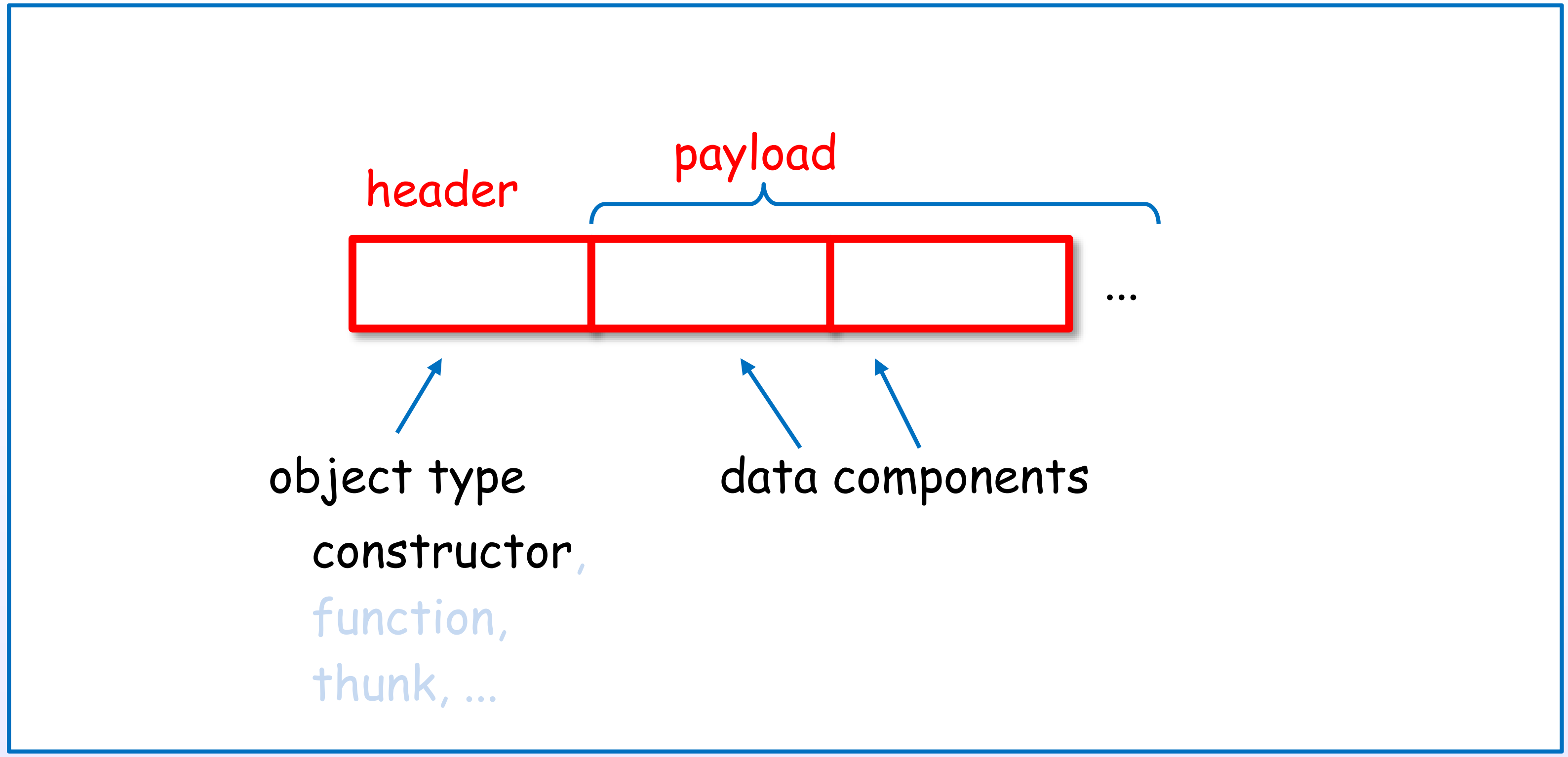
Internal representation of Constructors for data values



References : [H11], [H10], [H5], [H6], [H7]

Constructors are represented uniformly

GHC's internal representation

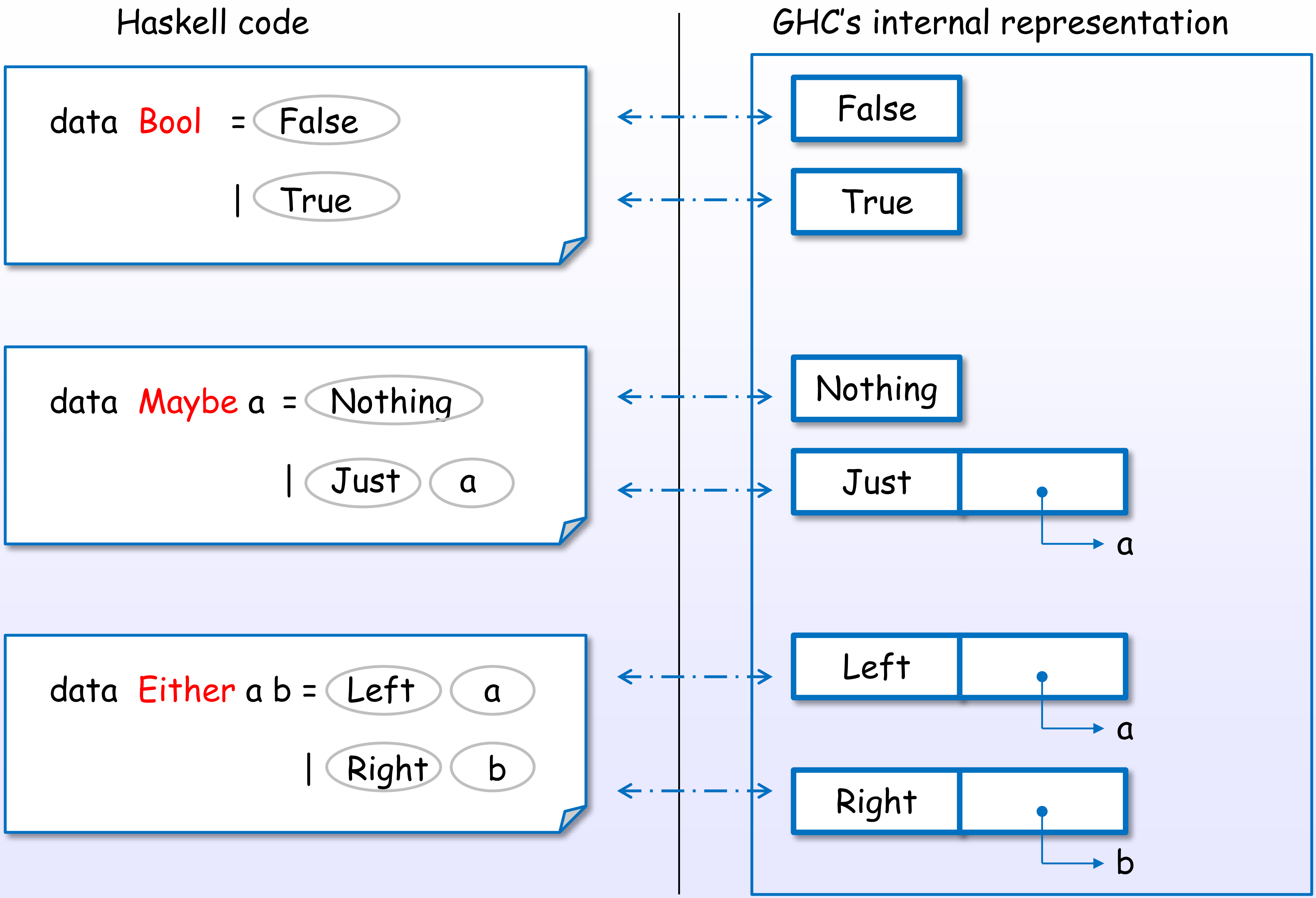


in heap memory, stack or static memory

A data value is represented with header(constructor) + payload(components).

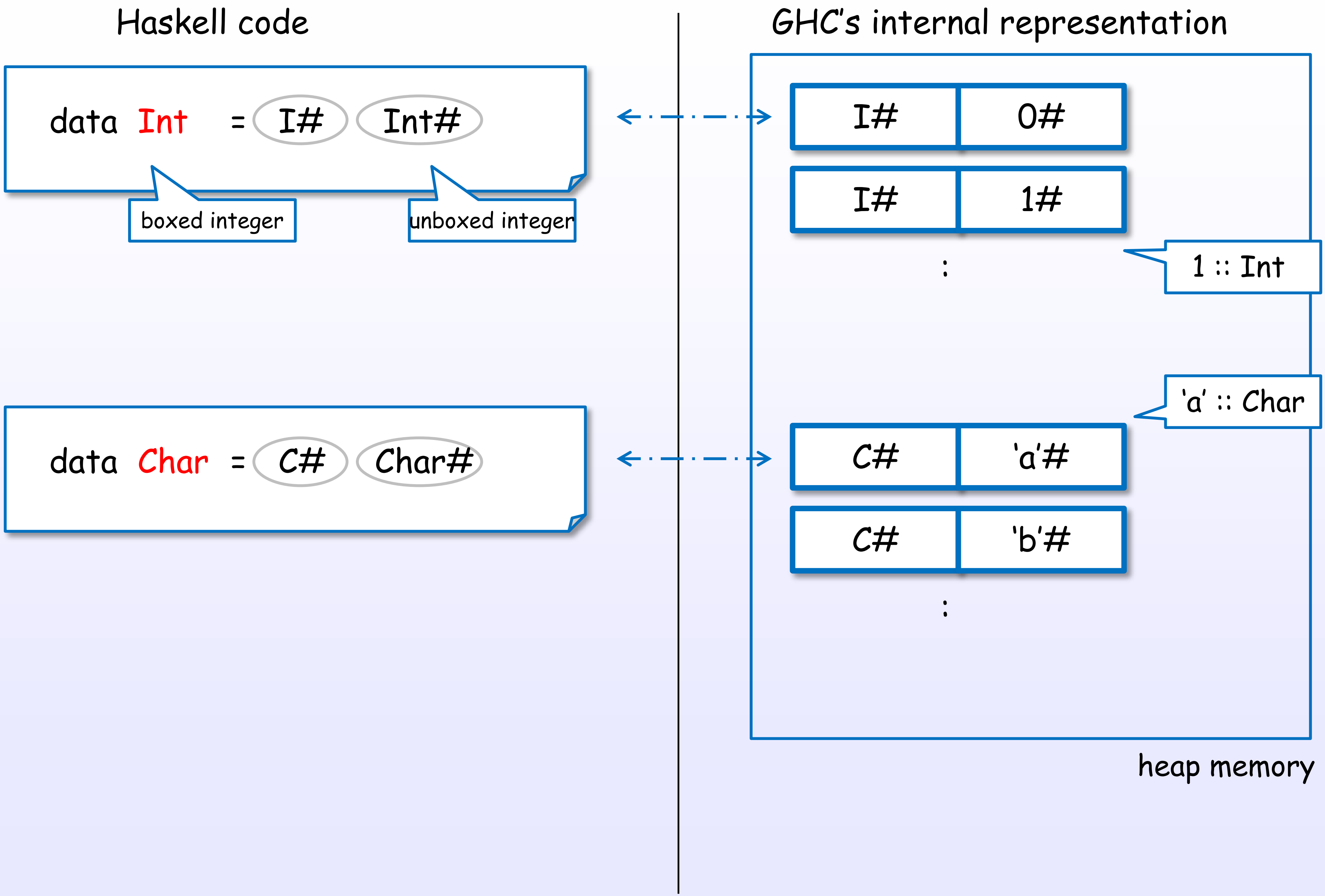
References : [H11], [H10], [H5], [H6], [H7], [D15]

Representation of various constructors



References : [H11], [H10], [H5], [H6], [H7]

Primitive data types are also represented with constructors



References : [H11], [H10], [H5], [H6], [H7]

List is also represented with constructors

List

```
[ 1, 2, 3 ]
```

syntactic desugar

```
1 : ( 2 : ( 3 : [] ) )
```

prefix notation by section

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

equivalent data constructor

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

constructor

References : [H11], [H10], [H5], [H6], [H7]

List is also represented with constructors

List

[1, 2, 3]

syntactic desugar

1 : (2 : (3 : []))

prefix notation by section

(:) 1 ((:) 2 ((:) 3 []))

equivalent data constructor

Cons 1 (Cons 2 (Cons 3 Nil))

type declaration

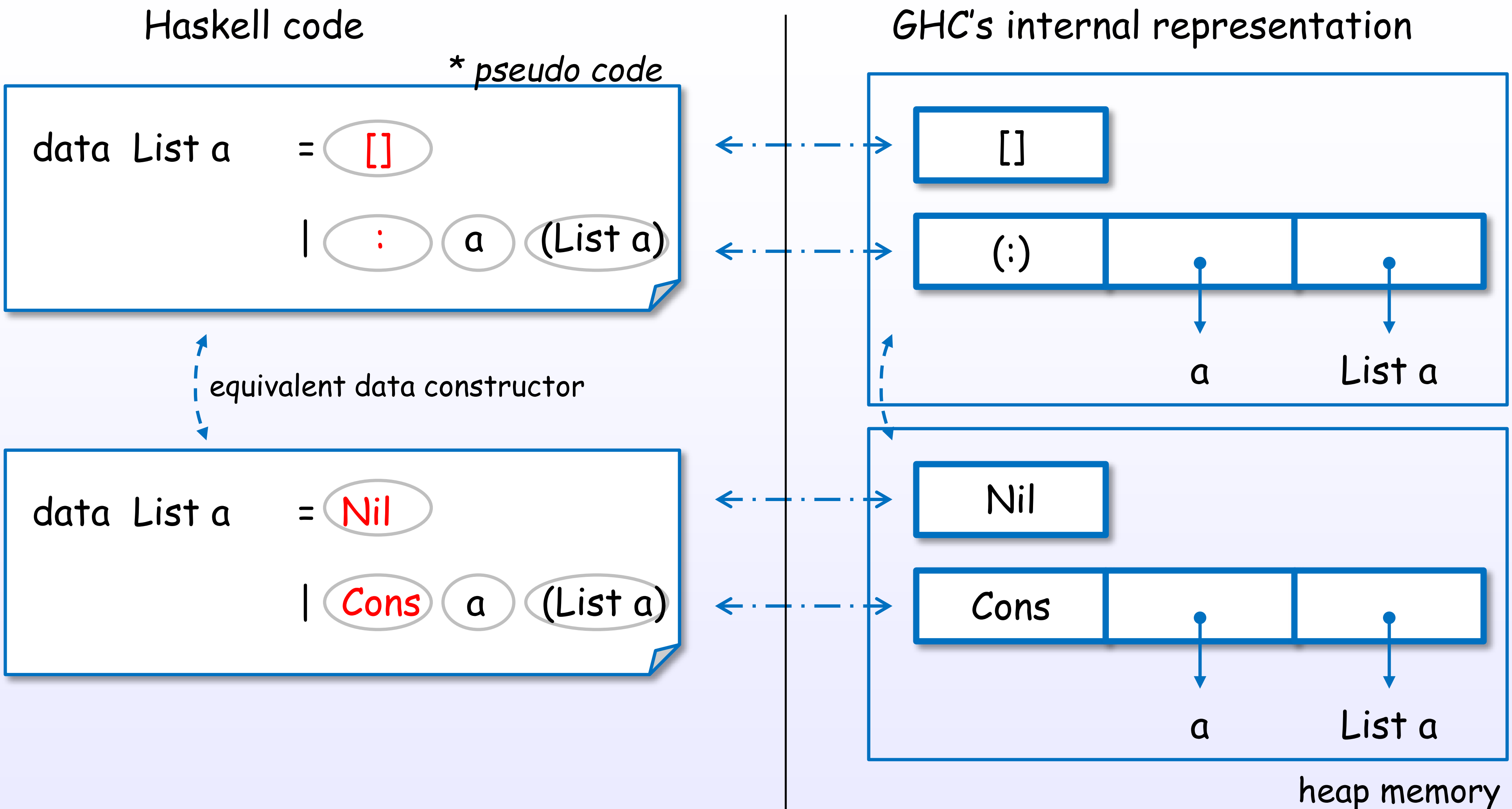
** pseudo code*

```
data  List a  =  []
              |  :  a  (List a)
```

```
data  List a  =  Nil
              |  Cons  a  (List a)
```

References : [H11], [H10], [H5], [H6], [H7]

List is also represented with constructors



References : [H11], [H10], [H5], [H6], [H7]

List is also represented with constructors

Haskell code

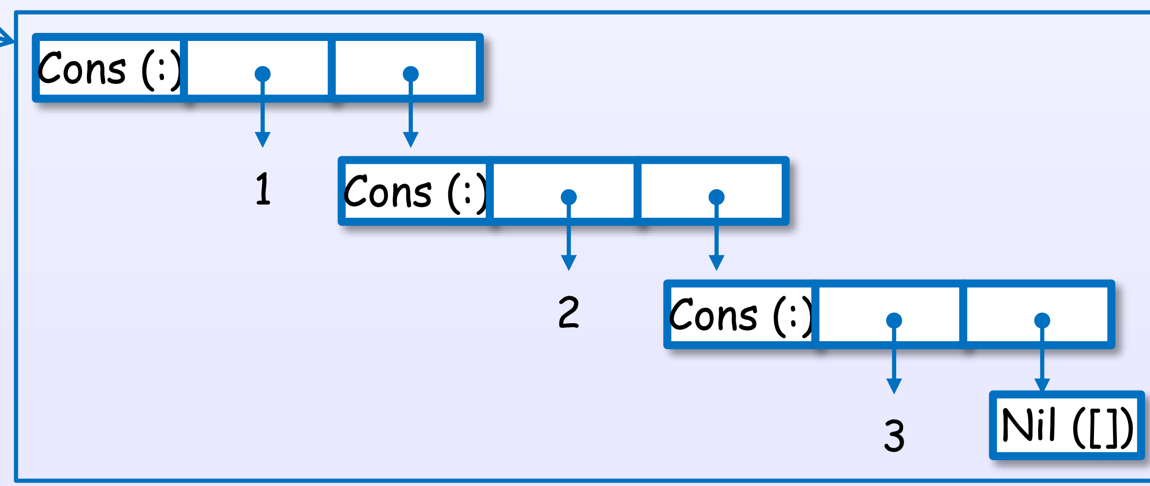
```
[ 1, 2, 3 ]
```

```
1 : ( 2 : ( 3 : [] ) )
```

```
(:) 1 ( (:) 2 ( (:) 3 [] ) )
```

```
Cons 1 ( Cons 2 ( Cons 3 Nil ) )
```

GHC's internal representation



References : [H11], [H10], [H5], [H6], [H7]

Tuple is also represented with constructor

Tuple (Pair)

```
( 7, 8 )
```

prefix notation by section

```
(,) 7 8
```

equivalent data constructor

```
Pair  7  8
```

constructor

type declaration

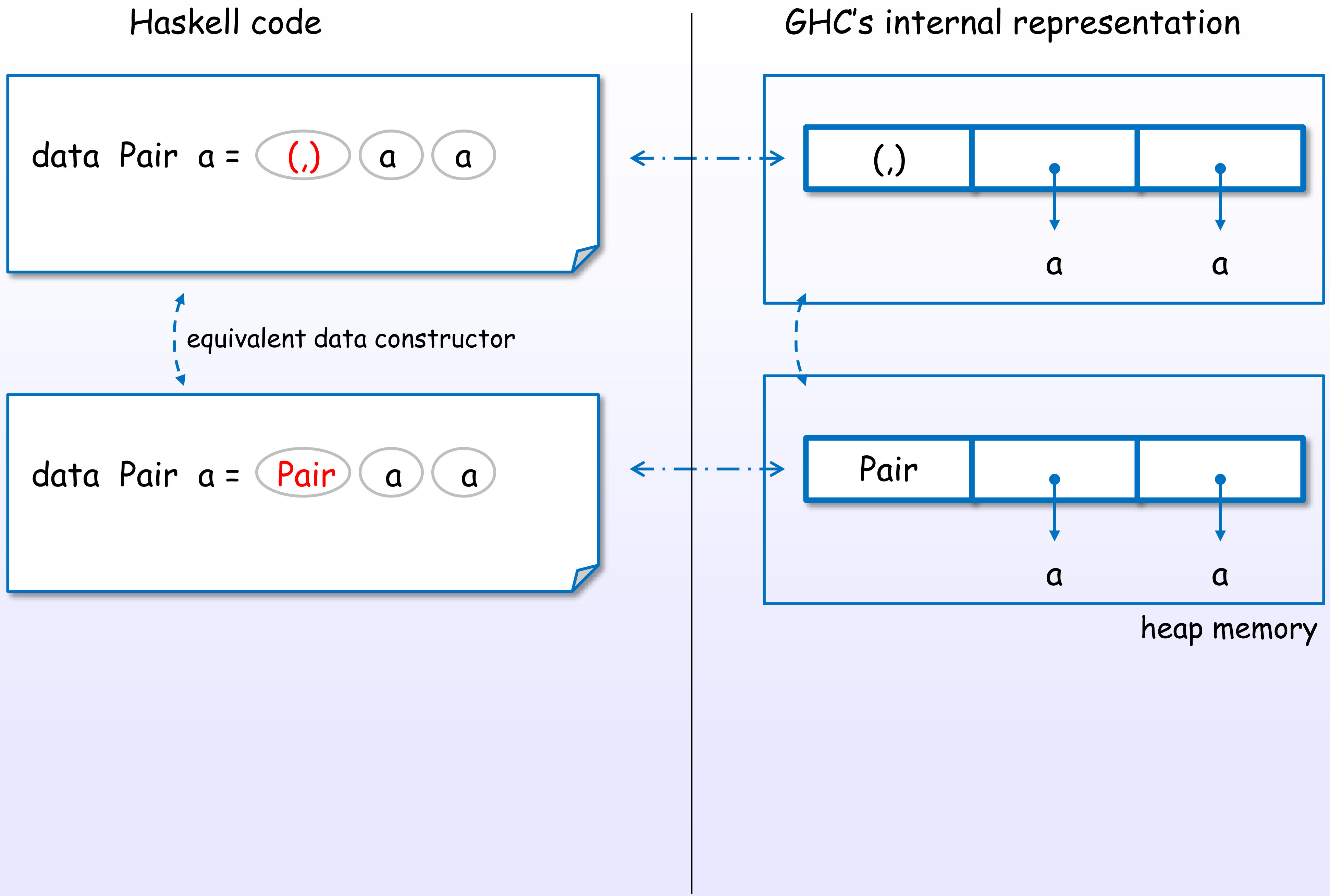
* *pseudo code*

```
data Pair a = (,) a a
```

```
data Pair a = Pair a a
```

References : [H11], [H10], [H5], [H6], [H7]

Tuple is also represented with constructor



References : [H11], [H10], [H5], [H6], [H7]

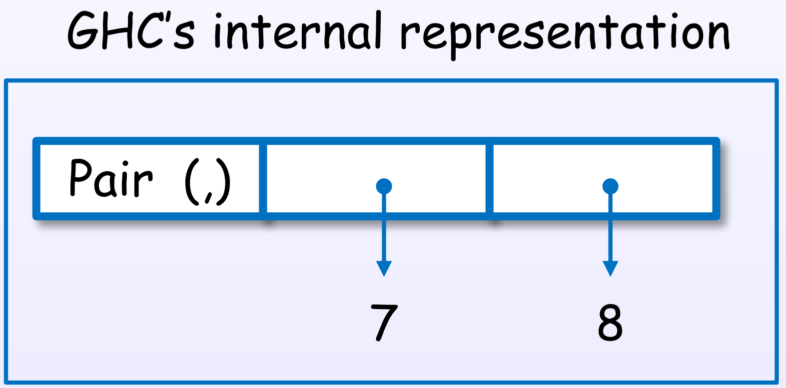
Tuple is also represented with constructor

Haskell code

```
( 7, 8 )
```

```
(,) 7 8
```

```
Pair  7  8
```

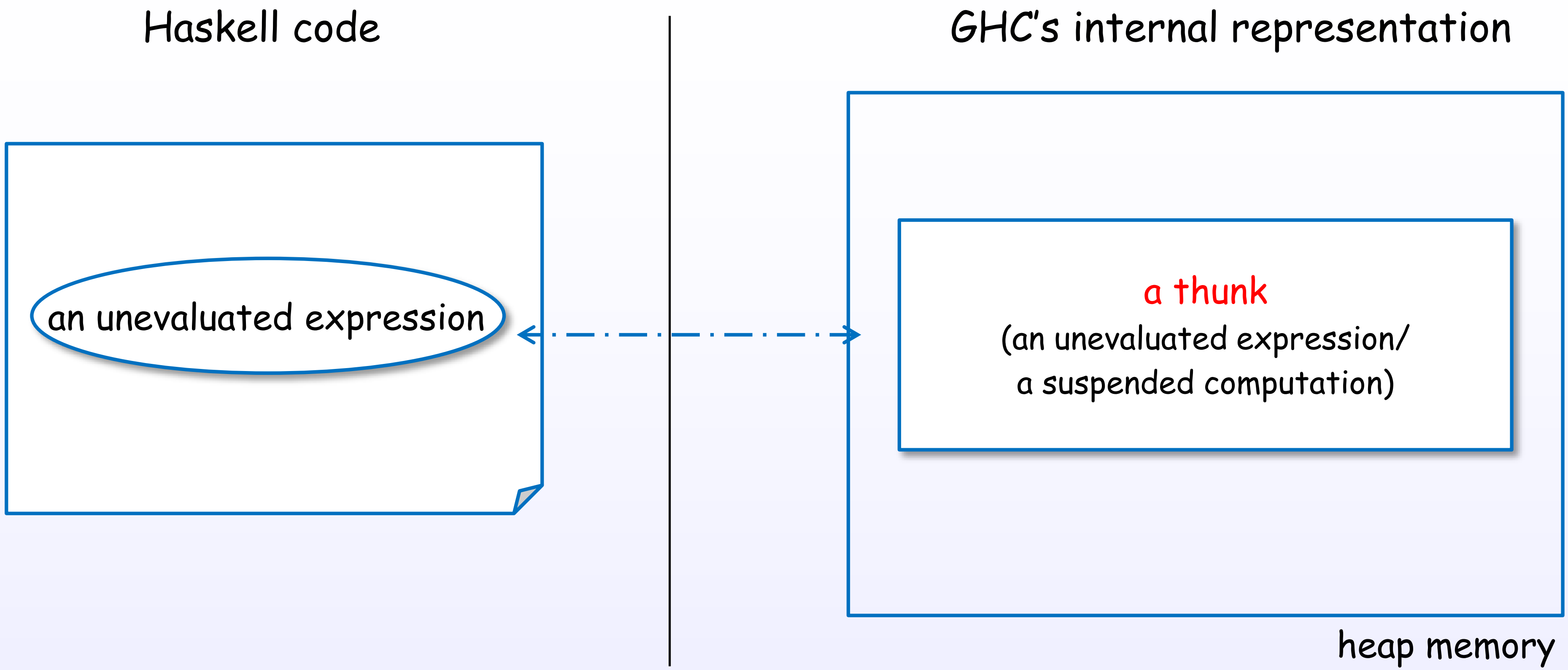


References : [H11], [H10], [H5], [H6], [H7]

3. Internal representation of expressions

Thunk

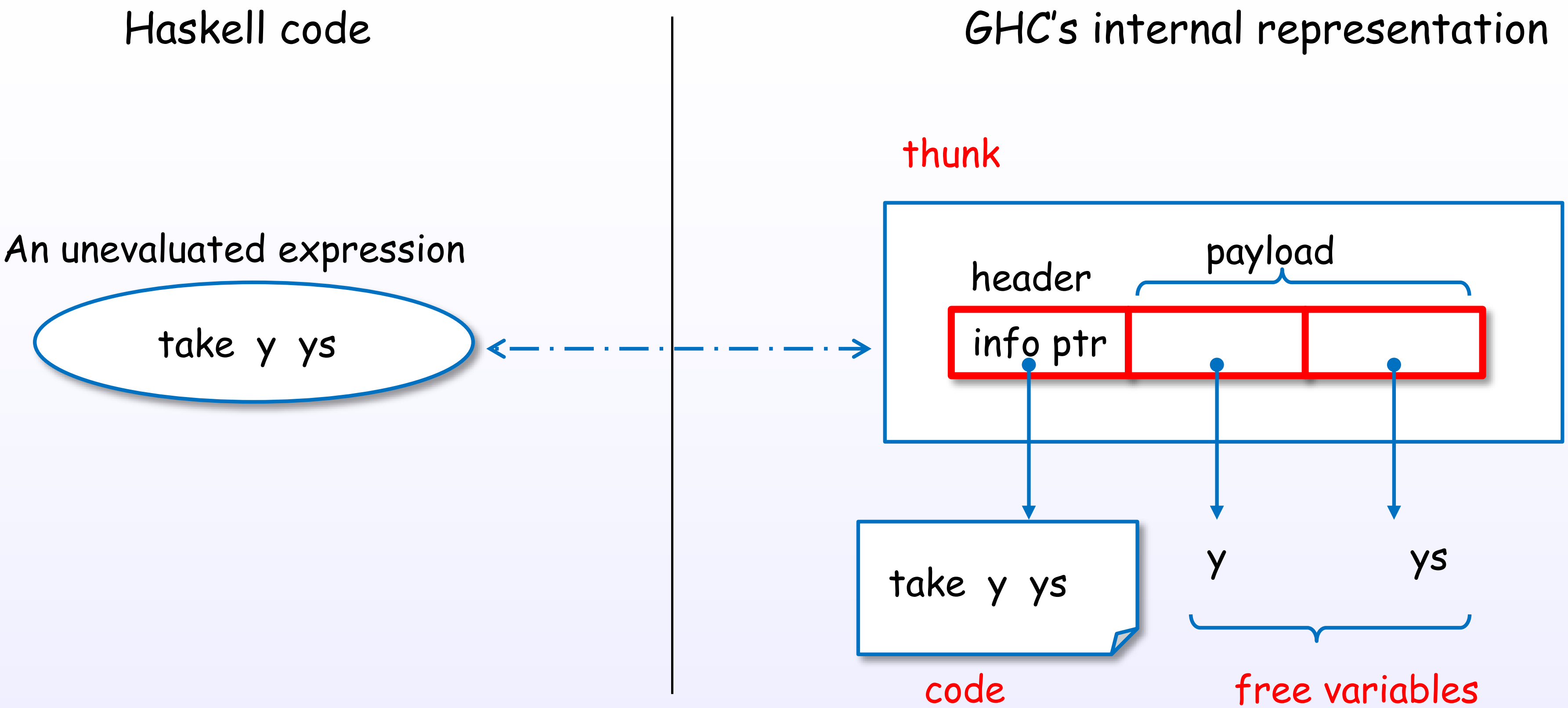
Thunk



A thunk is an unevaluated expression in heap memory.
A thunk is built to postpone the evaluation.

References : [B5] Ch.2, [D5], [W1], [H10], [H5], [D7]

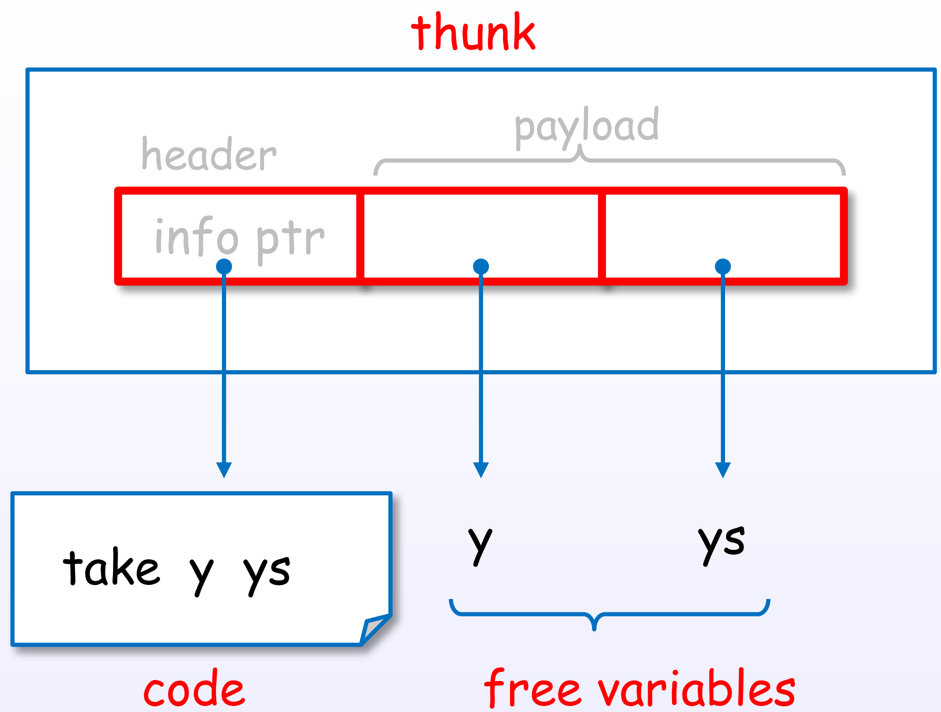
Internal representation of a thunk



A thunk is represented with header(code) + payload(free variables).

References : [H11], [H10], [D2], [H5], [H6], [H7], [B5] Ch.2, [D5], [W1]

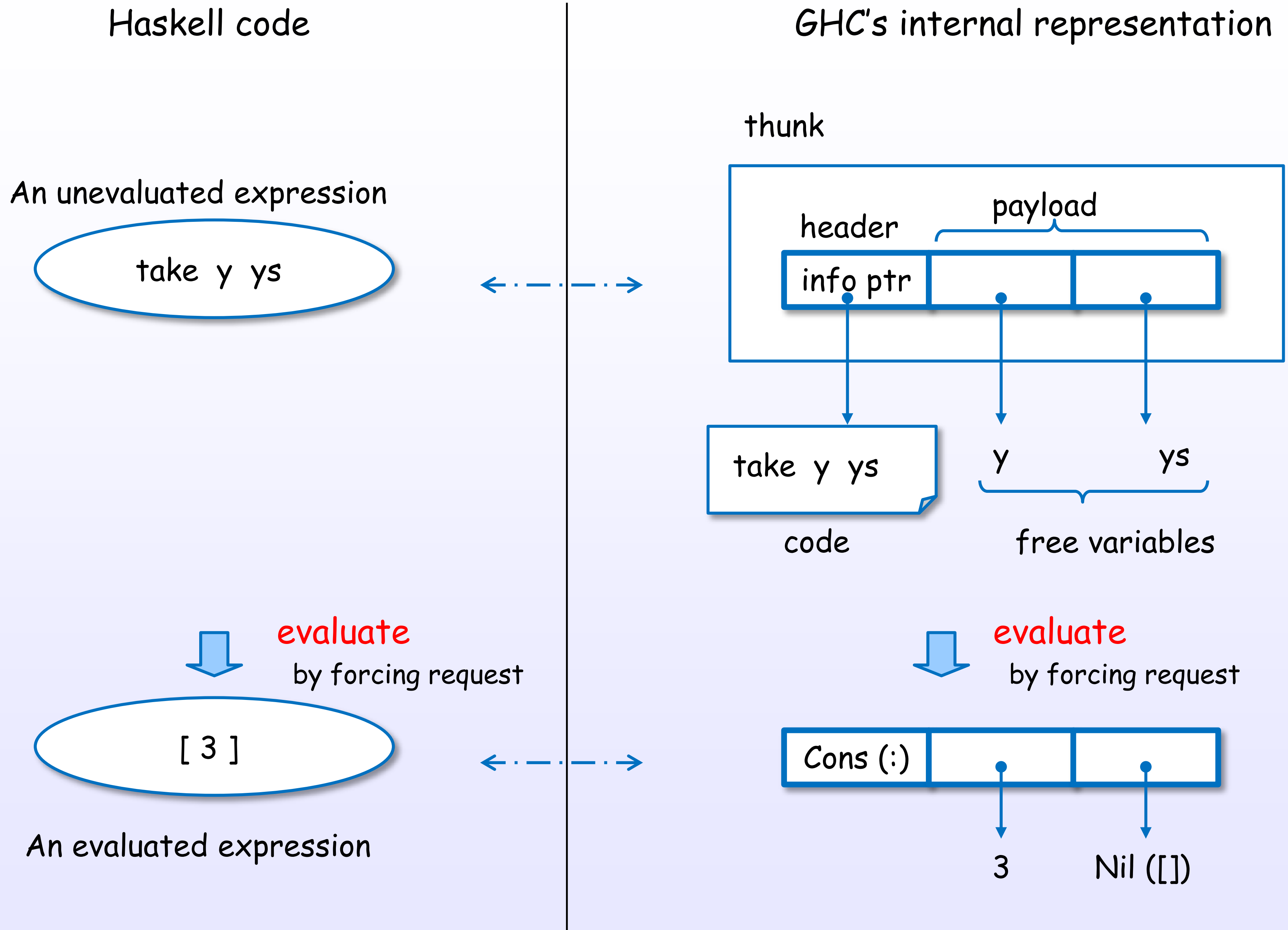
A thunk is a package



A thunk is a package of code + free variables.

References : [D2], [H11], [H10], [H5], [H6], [H7], [B5] Ch.2, [D5], [W1]

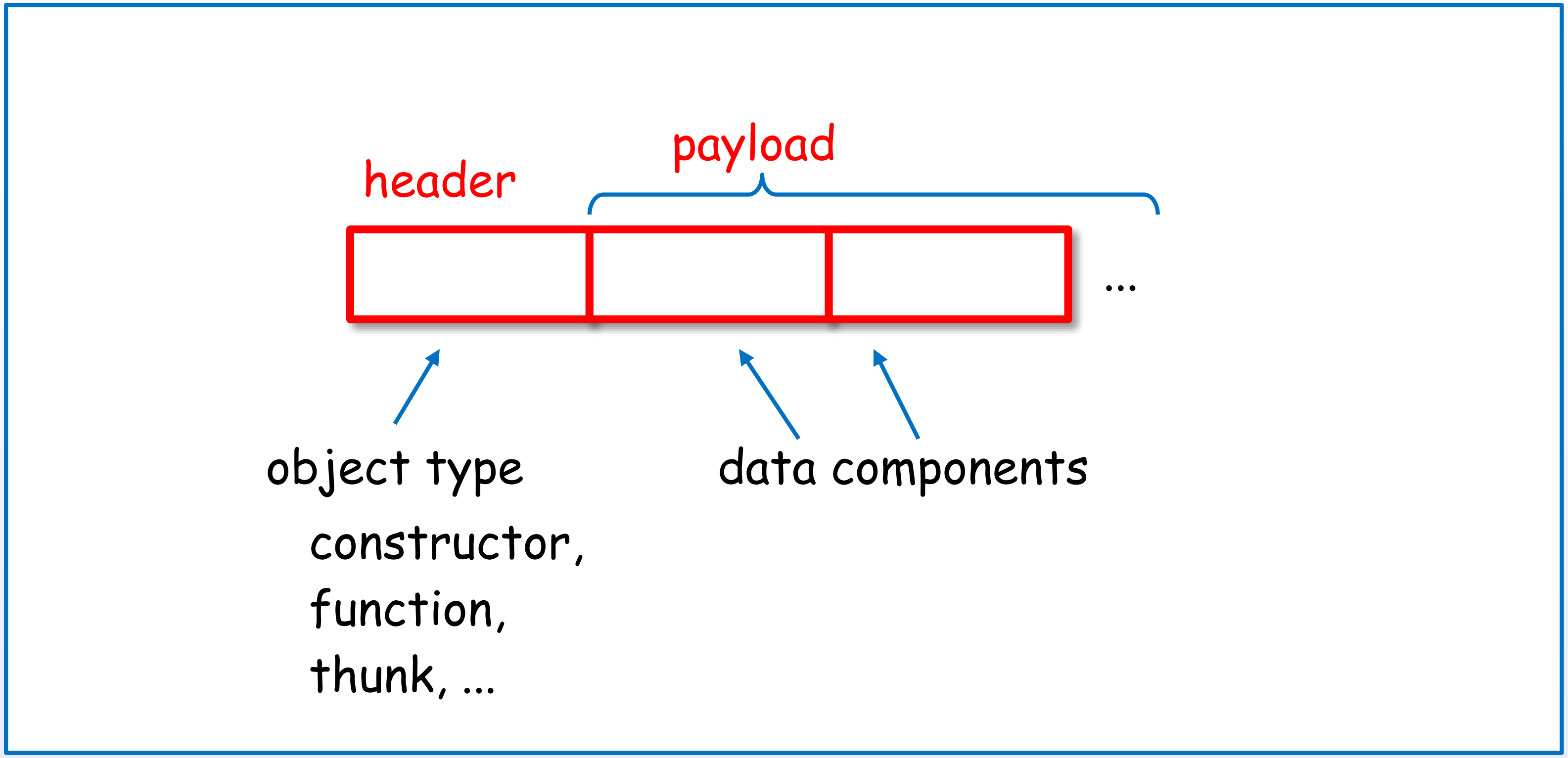
A thunk is evaluated by forcing request



References : [D7], [D2], [H11], [H10], [H5], [H6], [H7], [B5] Ch.2, [D5], [W1], [D15]

Uniform representation

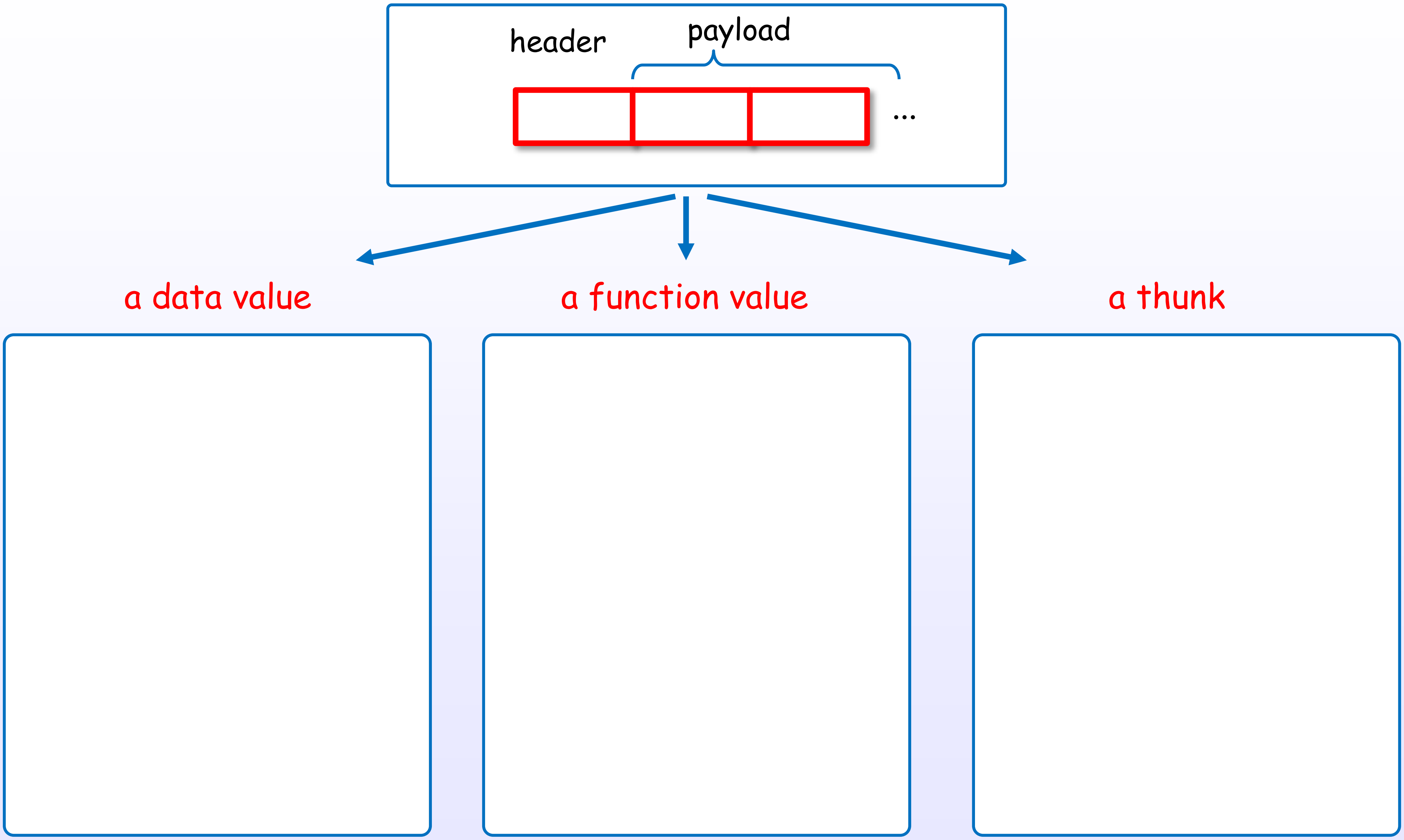
Every object is uniformly represented in memory



in heap memory, stack or static memory

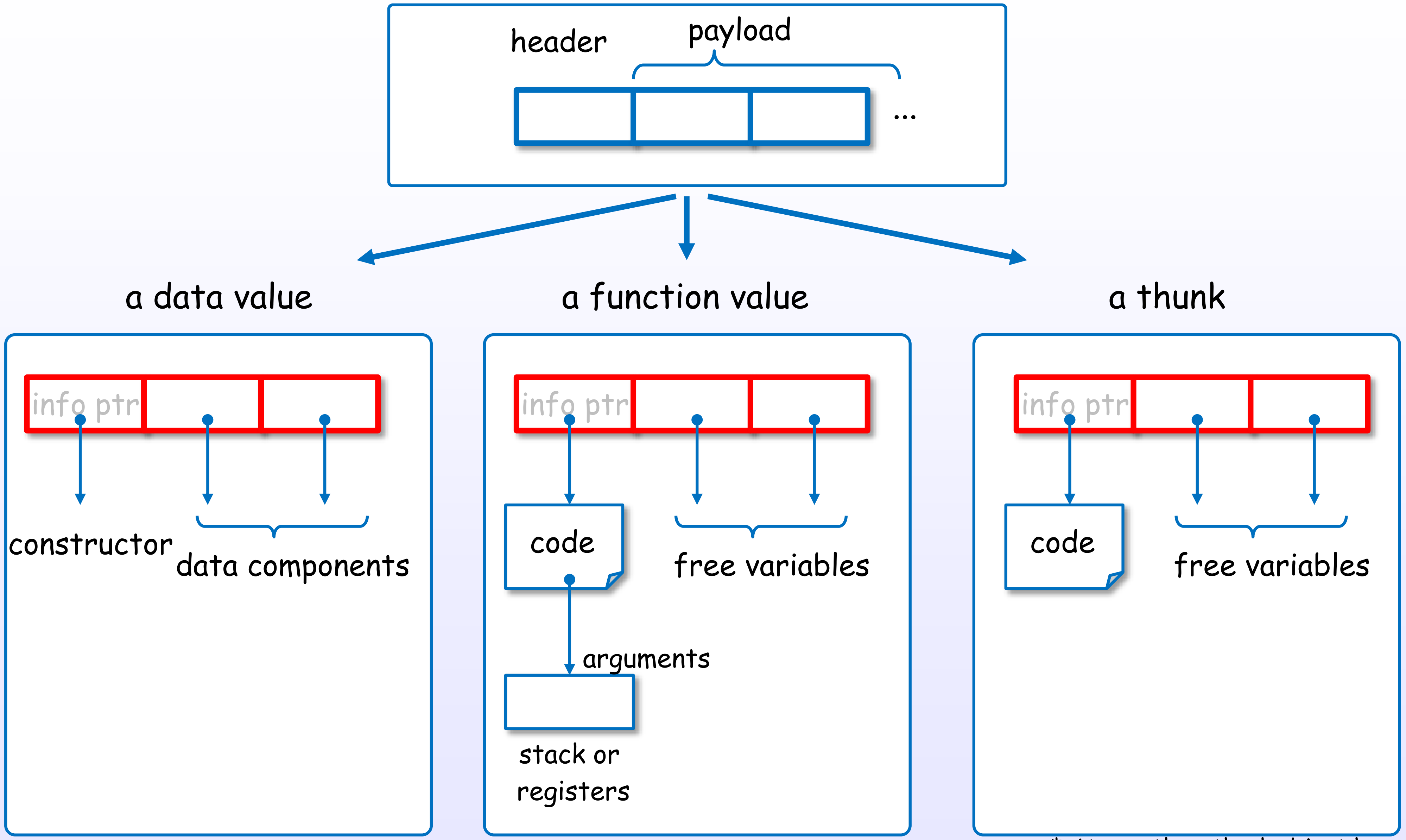
References : [H11], [H10], [H5], [H6], [H7], [D15]

Every object is uniformly represented in memory



References : [H11], [H10], [H5], [H6], [H7], [D15]

Every object is uniformly represented in memory



* At exactly, a thunk object has a reserved field in second.

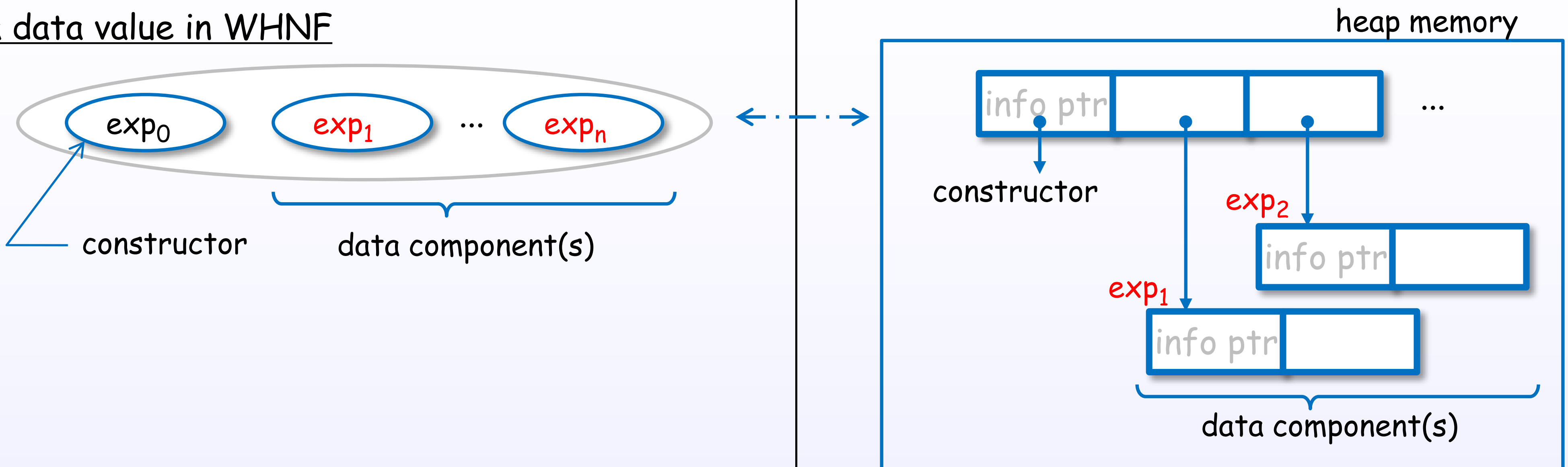
References : [H11], [H10], [H5], [H6], [H7], [D15]

WHNF

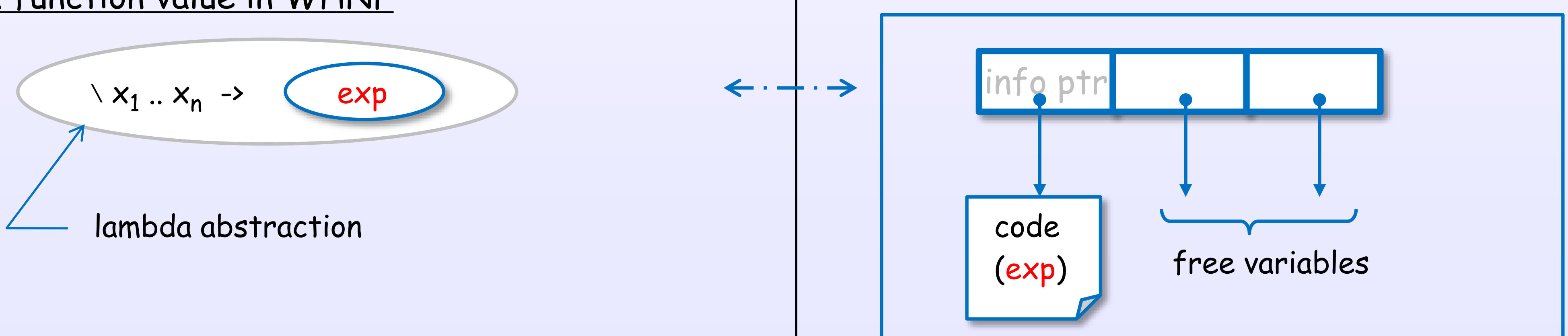
Internal representation of WHNF

Haskell code | GHC's internal representation

a data value in WHNF



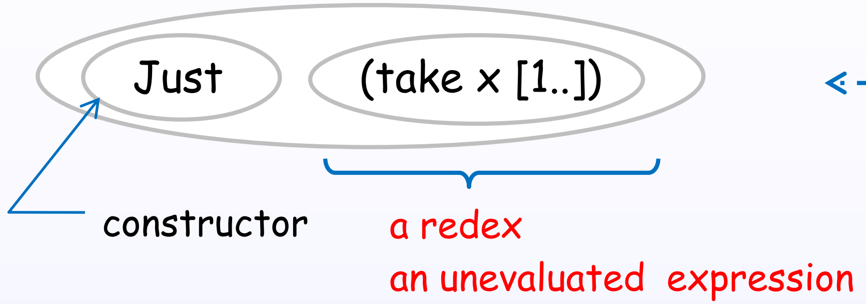
a function value in WHNF

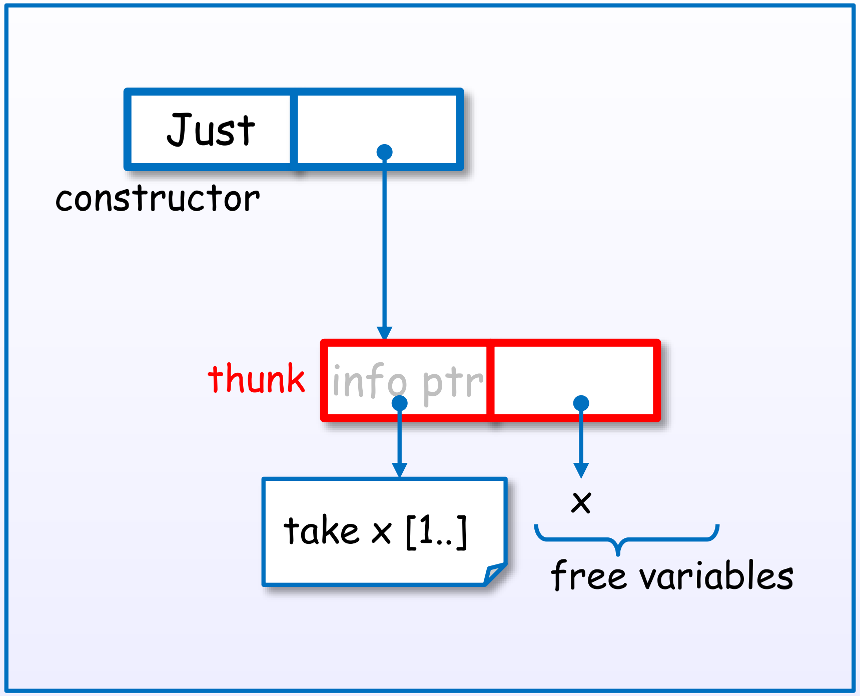


References : [H11], [H5], [H6], [H7], [H10]

Example of WHNF for a data value

Haskell code | GHC's internal representation



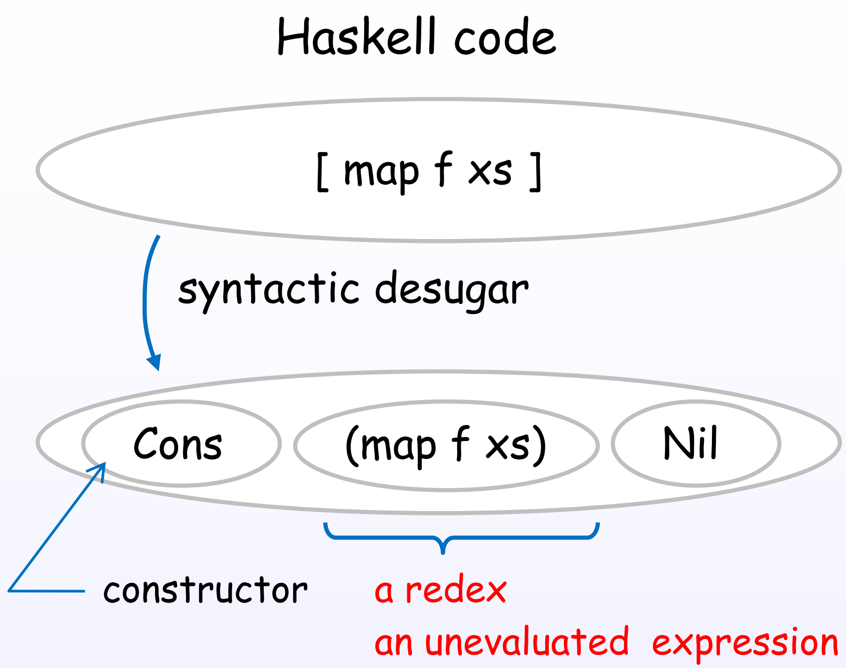


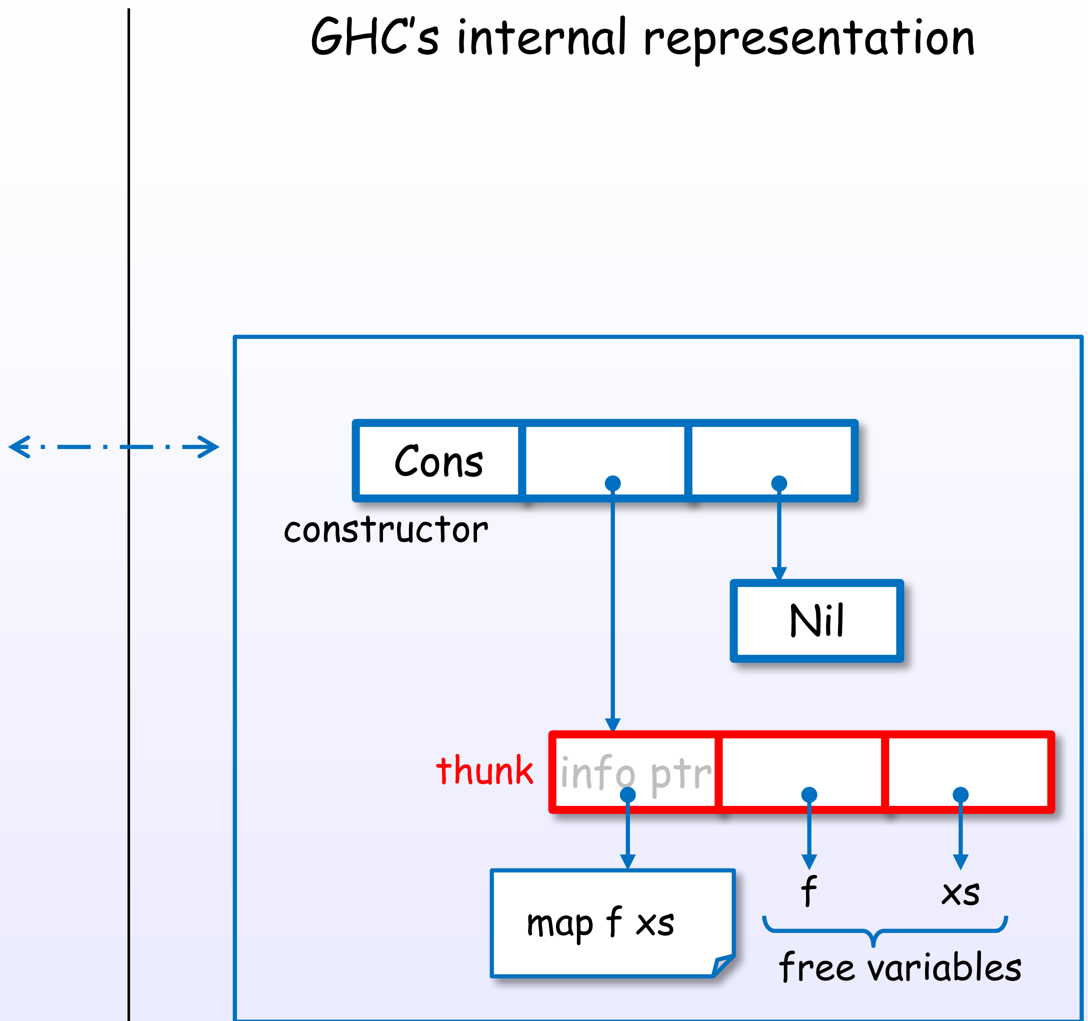
Constructors can contain unevaluated expressions by thunks.

Haskell's constructors are lazy constructors.

References : [H11], [H5], [H6], [H7], [H10]

Example of WHNF for a data value





References : [H11], [H5], [H6], [H7], [H10]

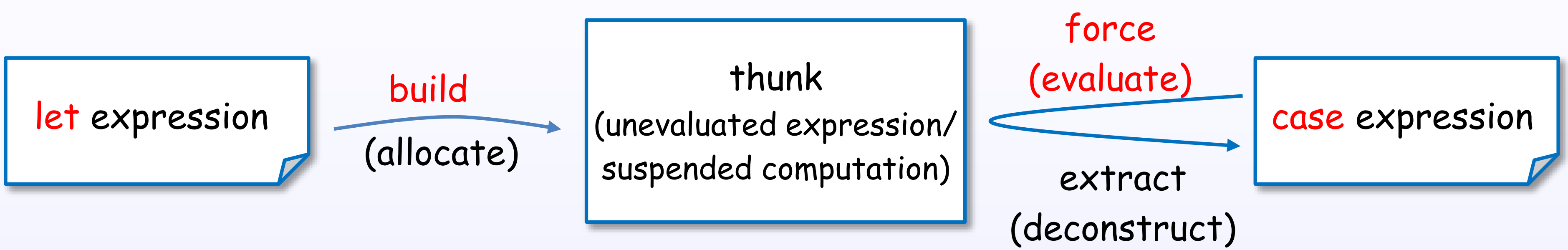
3. Internal representation of expressions

let, case expression

let, case expression

let and case expressions are special role in the evaluation

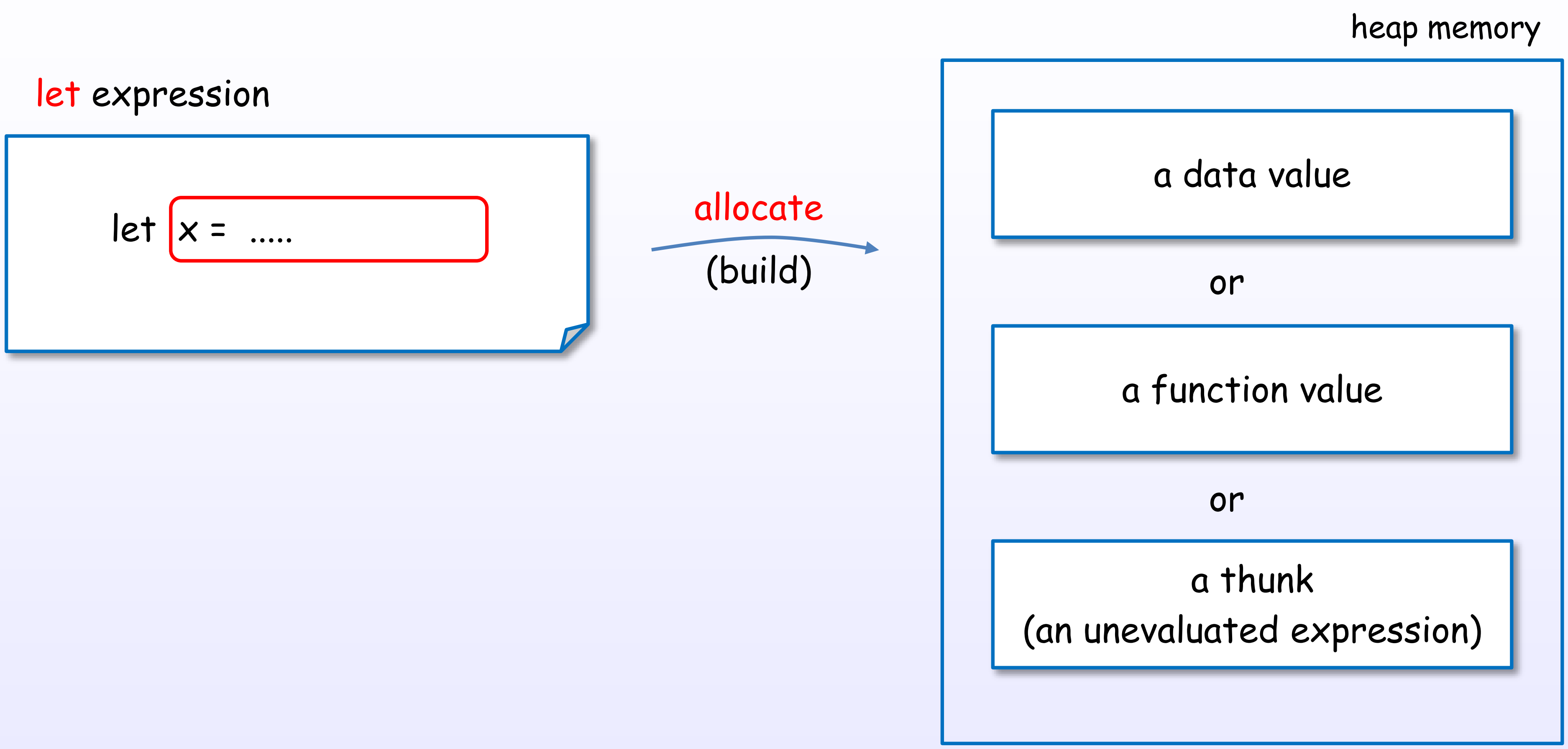
let/case expressions and thunk



A let expression may build a thunk.
A case expression evaluates (forces) and deconstructs the thunk.

References : [H5], [H6], [H7], [H10]

A let expression may allocates a heap object



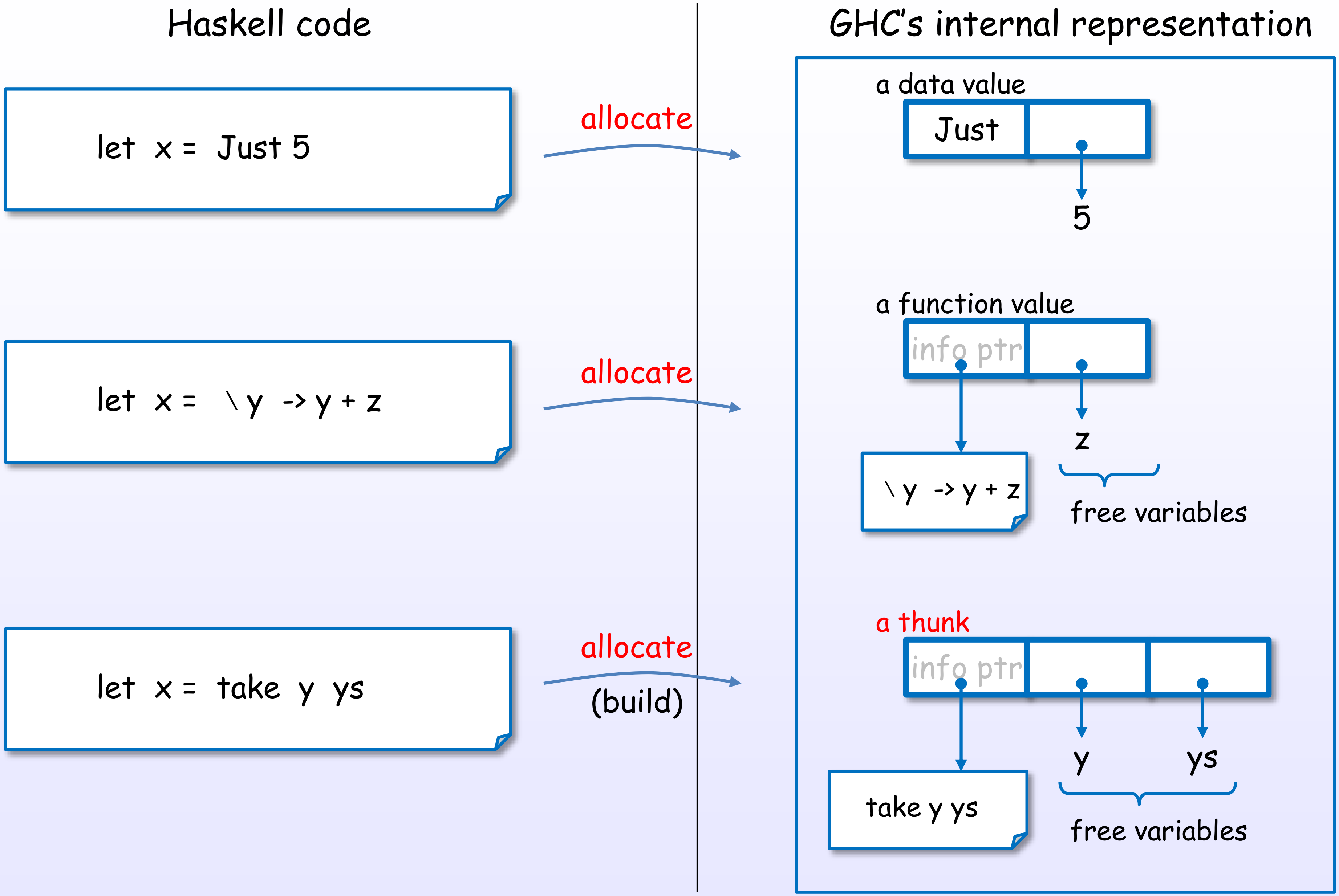
A let expression may allocates an object in the heap.

(If GHC can optimize it, the let expression may not allocate.)

* At exactly, STG language's let expression rather than Haskell's let expression

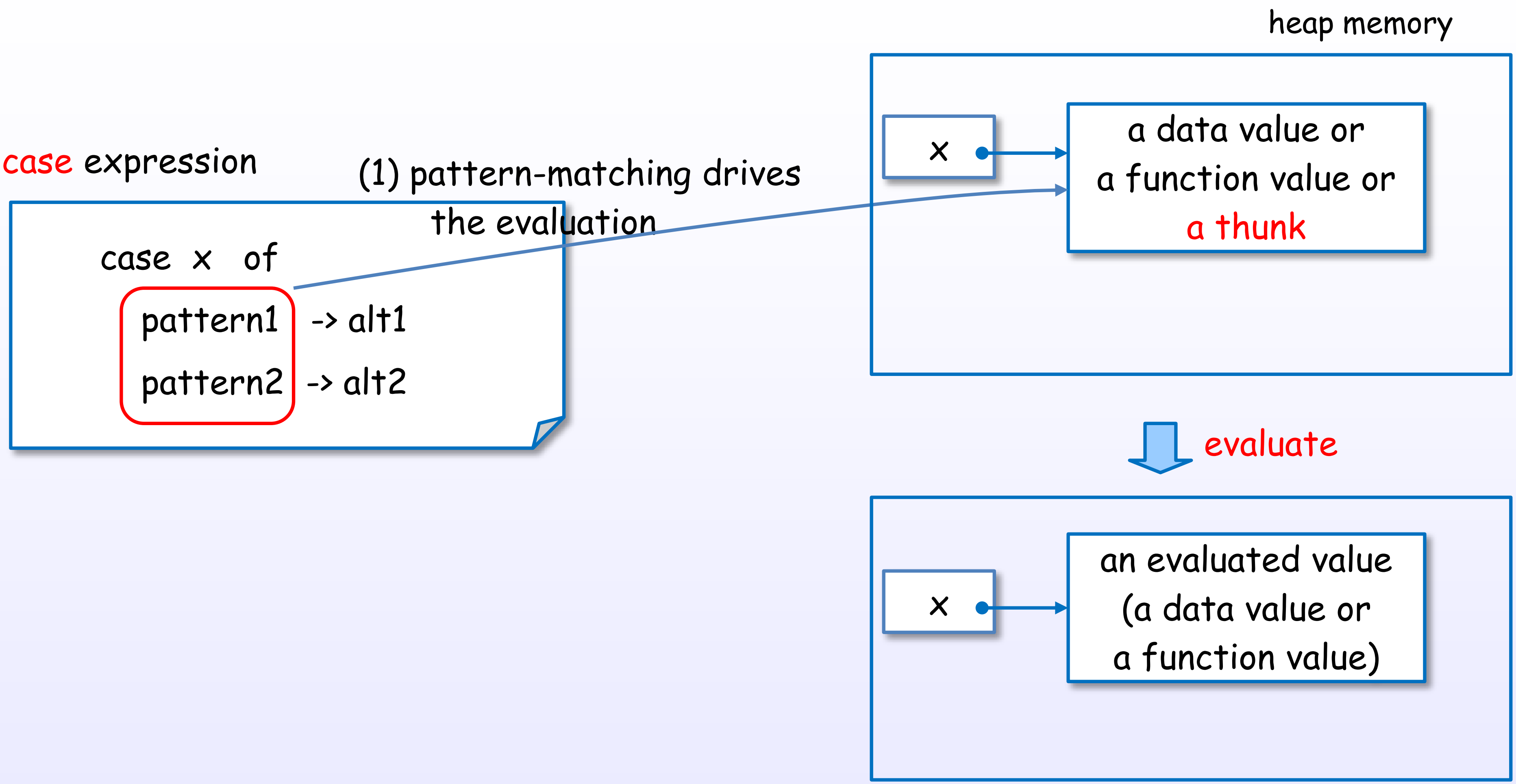
References : [H5], [H6], [H7], [H10]

Example of let expressions



References : [H5], [H6], [H7], [H10]

A case expression evaluates a subexpression

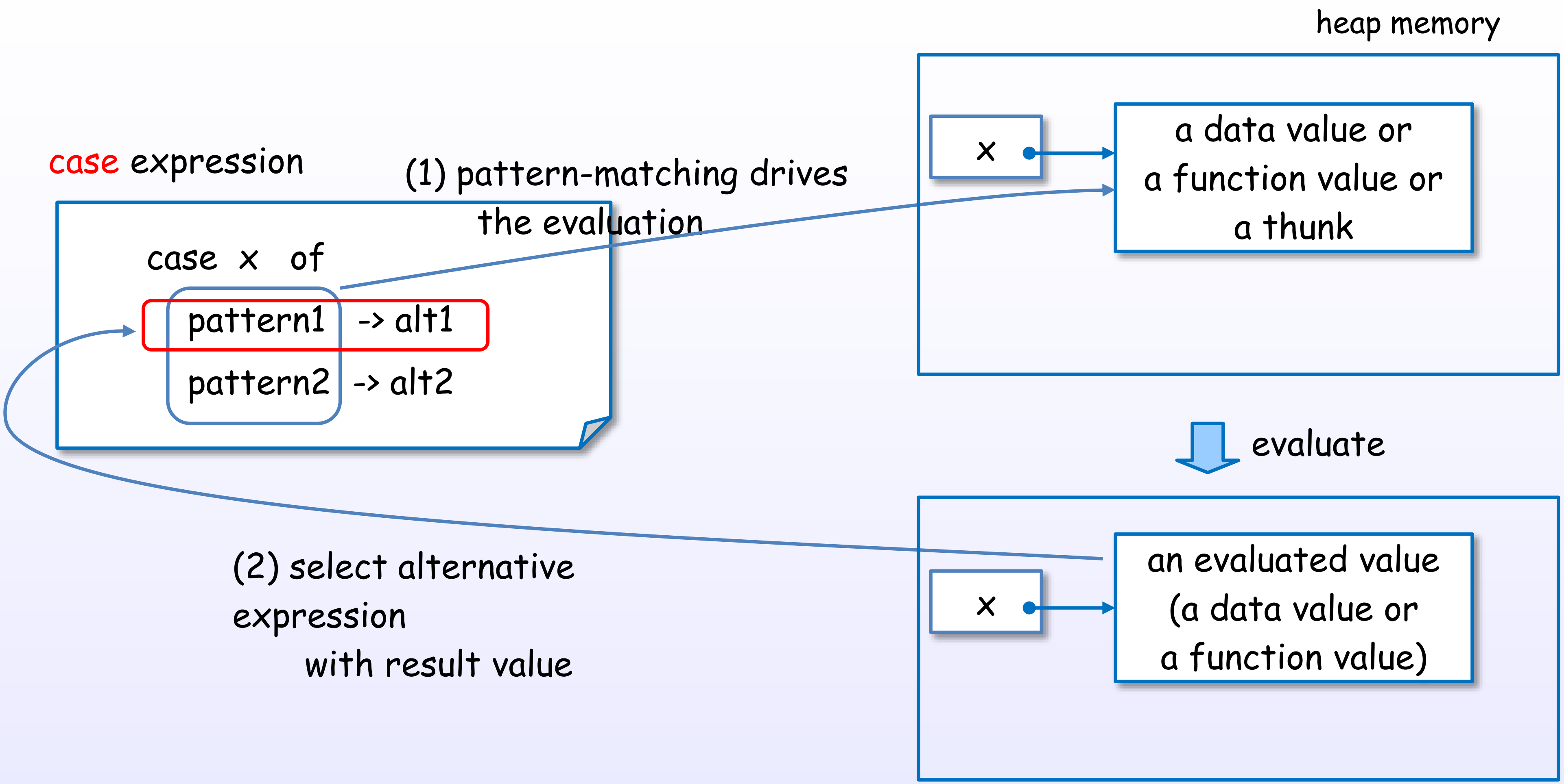


Pattern-matching drives the evaluation.

* At exactly, STG language's case expression rather than Haskell's case expression

References : [H5], [H6], [H7], [H10], [D2]

A case expression also perform case analysis

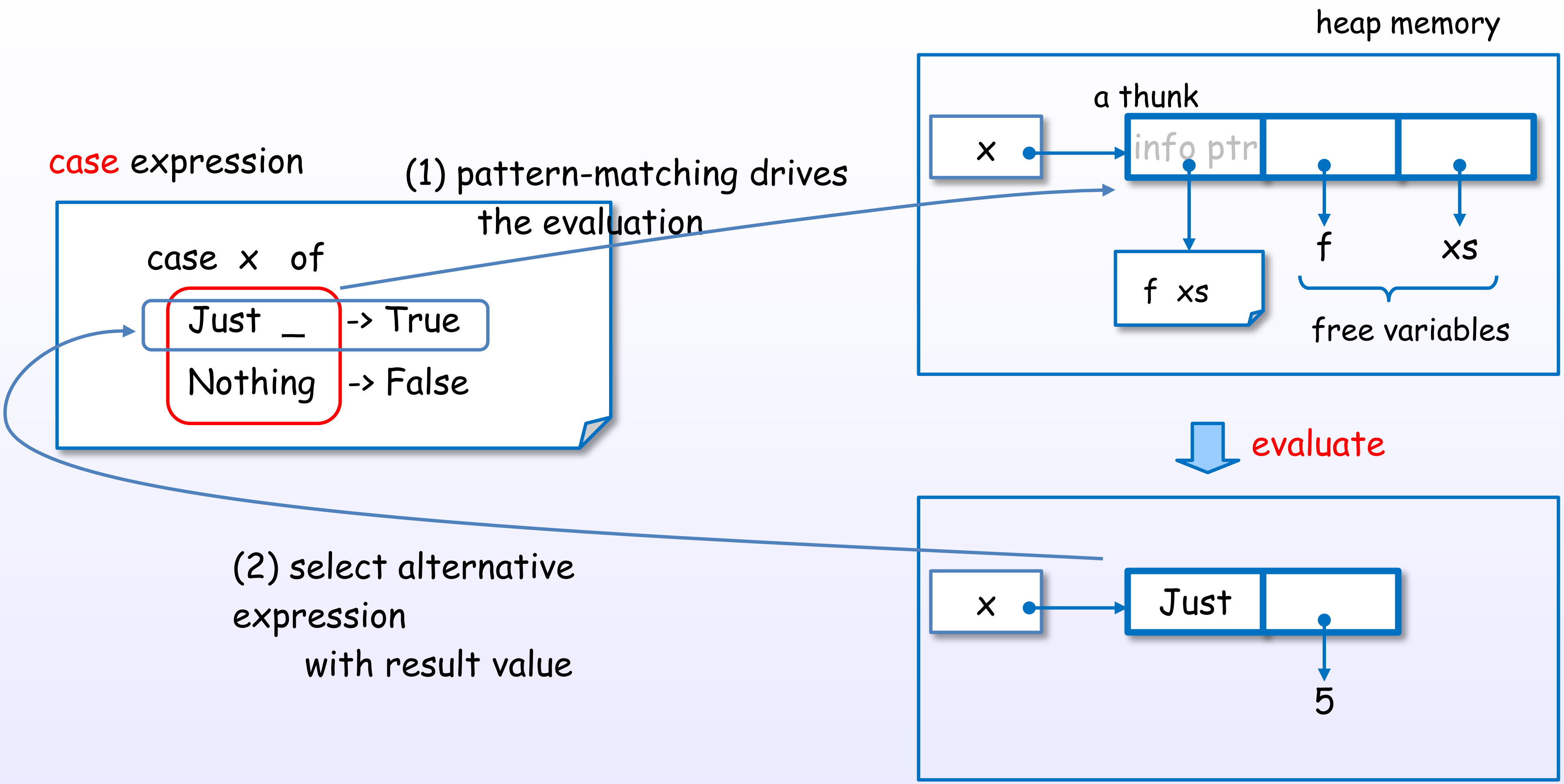


A case expression evaluates a subexpression
and optionally performs case analysis on its value.

* At exactly, STG language's case expression rather than Haskell's case expression

References : [H5], [H6], [H7], [H10], [D2]

Example of a case expression



A case expression's pattern-matching says "I need the value".

References : [H5], [H6], [H7], [H10], [D2]

Pattern-matching in function definition

pattern-matching in function definition

```
f  Just _   =  True
f  Nothing  =  False
```

syntactic desugar

pattern-matching in case expression

```
f x = case x of
        Just _   -> True
        Nothing  -> False
```

A function's pattern-matching is syntactic sugar of case expression.

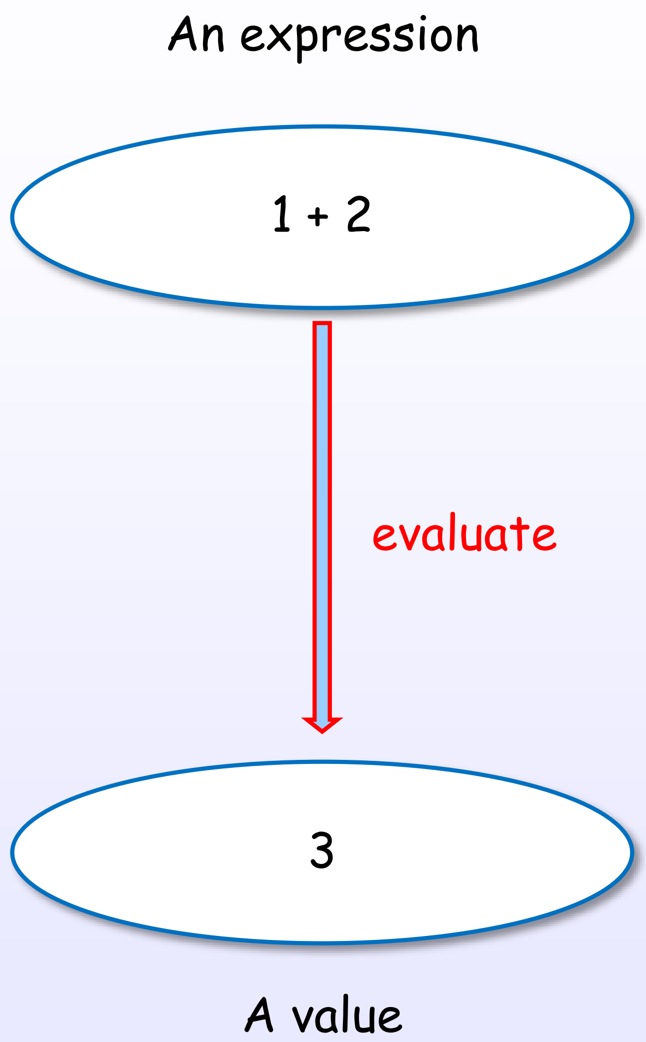
A function's pattern-matching also drives the evaluation.

References : [H1]

4. Evaluation

Evaluation strategies

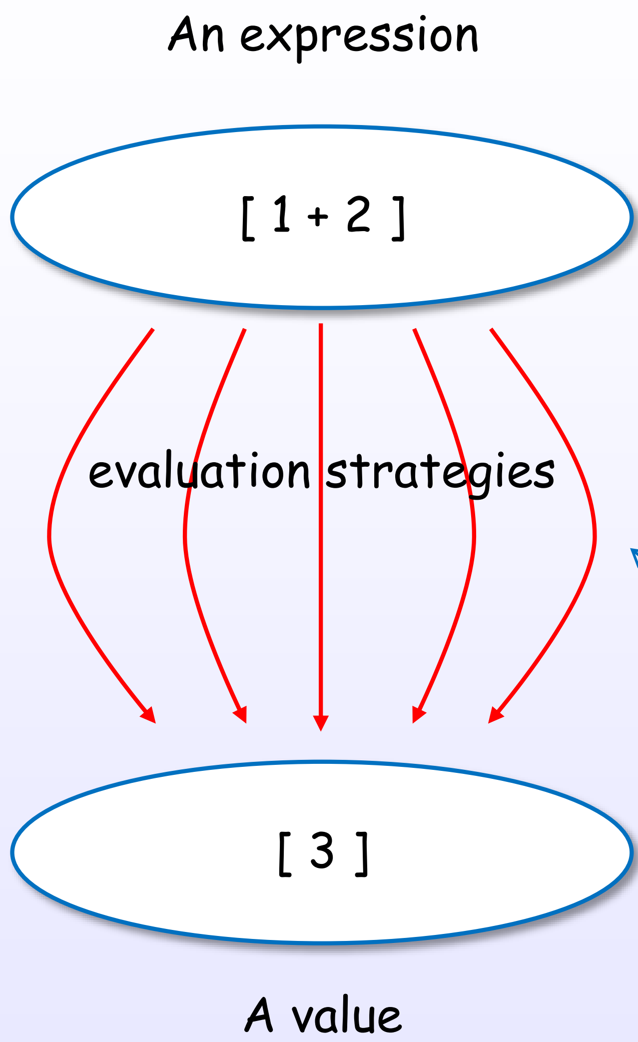
Evaluation



The evaluation produces a value from an expression.

References : [B1] Ch.1, [B2] Ch.2, [H1] Ch.1, [B6] Ch.3

There are many evaluation approaches



- Strict, Non-strict evaluation
- Eager, Lazy evaluation
- Call-by-value, Call-by-name, Call-by-need, …
- Innermost, Outermost
- Normal order, Application order
- …

References : [B3] Ch.2, 7, [B6] Ch.3, [D1]

Evaluation concept layer

Denotational semantics

Operational semantics
(Evaluation strategies / Reduction strategies)

Implementation techniques

Evaluation layer for GHC's Haskell

Denotational semantics

- Strict semantics
- Non-strict semantics

Operational semantics
(Evaluation strategies/ Reduction strategies)

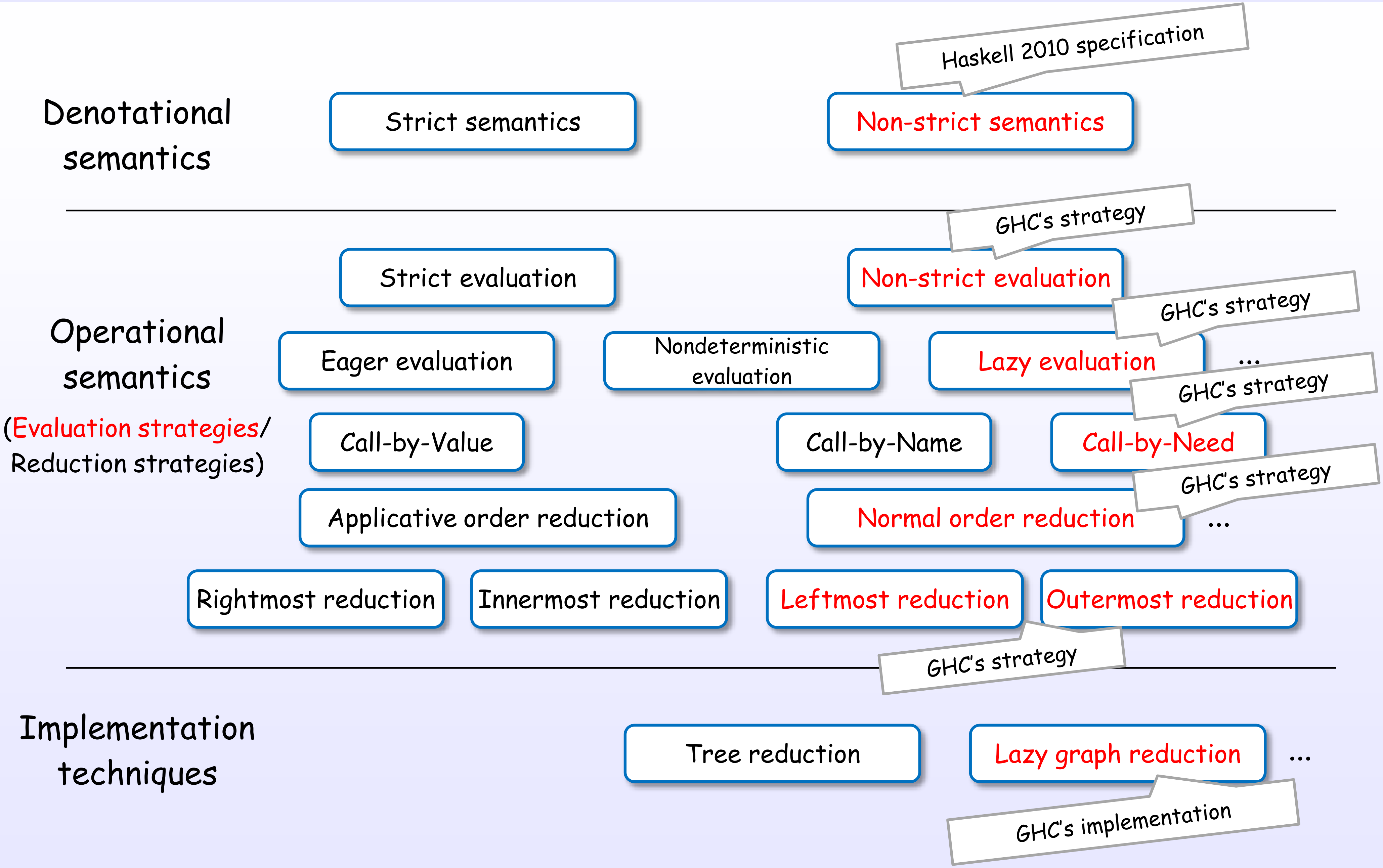
- Strict evaluation
- Non-strict evaluation
- Eager evaluation
- Nondeterministic evaluation
- Lazy evaluation
- …
- Call-by-Value
- Call-by-Name
- Call-by-Need
- …
- Applicative order reduction
- Normal order reduction
- …
- Rightmost reduction
- Innermost reduction
- Leftmost reduction
- Outermost reduction

Implementation techniques

- Tree reduction
- Lazy graph reduction
- …

References : [D3], [D1], [D2], [D5], [D4], [B2] Ch.7, [B3] Ch.8, [B6] Ch.5, [W1], [W2], [W3], [B7], [B8]

Evaluation layer for GHC's Haskell



References : [D3], [D1], [D2], [D5], [D4], [B2] Ch.7, [B3] Ch.8, [B6] Ch.5, [W1], [W2], [W3], [B7], [B8]

Evaluation strategies

Each evaluation strategy decides how to operate the evaluation, about ...

ordering,
region,
trigger condition,
termination condition,
re-evaluation, ...

References : [D3], [D1], [D2], [D5], [D4], [B2] Ch.7, [B3] Ch.8, [B6] Ch.5, [W1], [W2], [W3], [B7], [B8]

One of the important points is the order

which first?

apply

function arguments

apply first

lazy evaluation,
call-by-name,
call-by-need,
outermost reduction,
normal order reduction

argument first

eager evaluation,
call-by-value,
innermost reduction,
applicative order reduction

References : [D3], [D1], [D2], [D5], [D4], [B2] Ch.7, [B3] Ch.8, [B6] Ch.5, [W1], [W2], [W3], [B7], [B8]

Simple example of typical evaluations

default
C, Java, JavaScript, Python, OCaml, Scheme, ...

<u>call-by-value</u>

square (1 + 2)

argument evaluation first

default
Haskell (GHC), ...

<u>call-by-need</u>

square (1 + 2)

apply first

References : [B2] Ch.7, [B3] Ch.8, [D4], [B6] Ch.5

Simple example of typical evaluations

call-by-value

square (1 + 2)

⇩

square (3)

evaluation is performed

⇩

3 * 3

⇩

9

call-by-need

square (1 + 2)

⇩

(1 + 2) * (1 + 2)

evaluation is delayed !

⇩

(3) * (3)

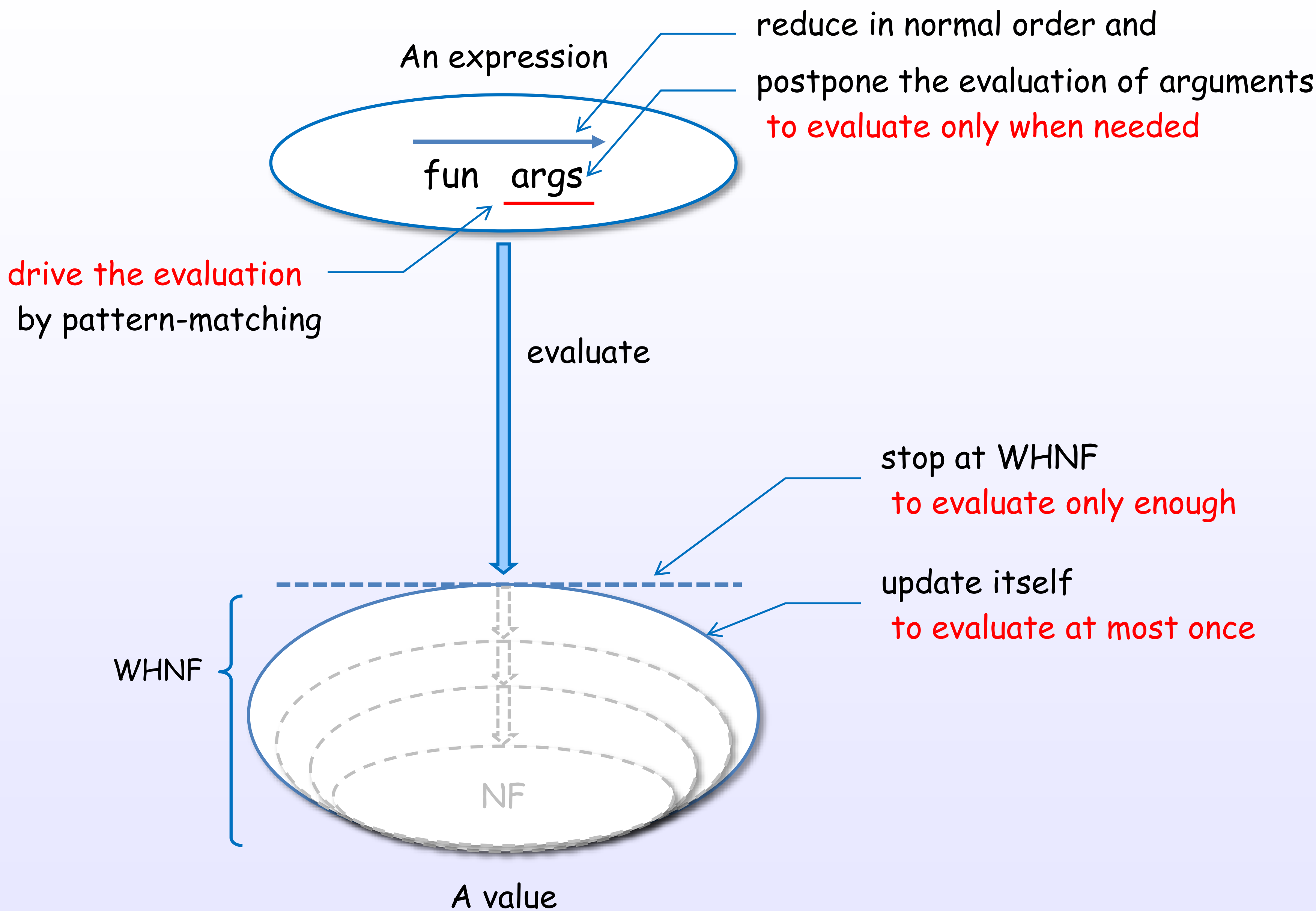
⇩

9

References : [B2] Ch.7, [B3] Ch.8, [D4], [B6] Ch.5

Evaluation in Haskell (GHC)

Key concepts of GHC's lazy evaluation



References : [H4] Ch.11, 12, [H5], [H6], [D2]

Postpone the evaluation of arguments

Haskell code

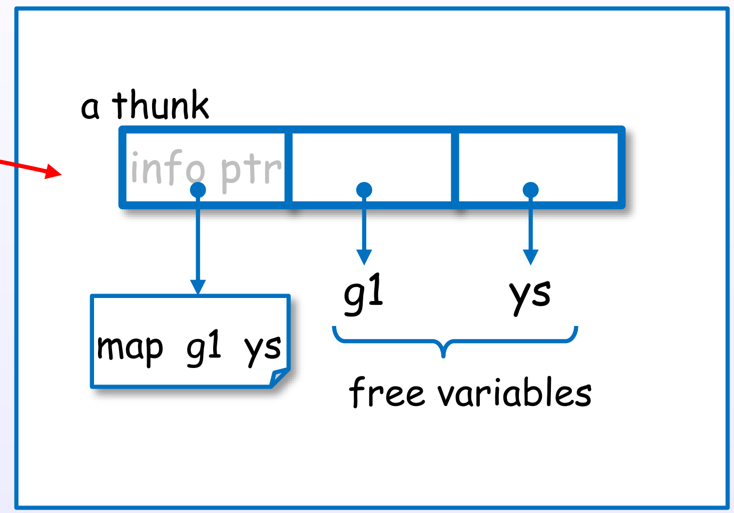
```
fun  (map  g1  ys)
```

internal translation

```
let  thunk0 = map  g1  ys
in  fun  thunk0
```

postpone

(build)



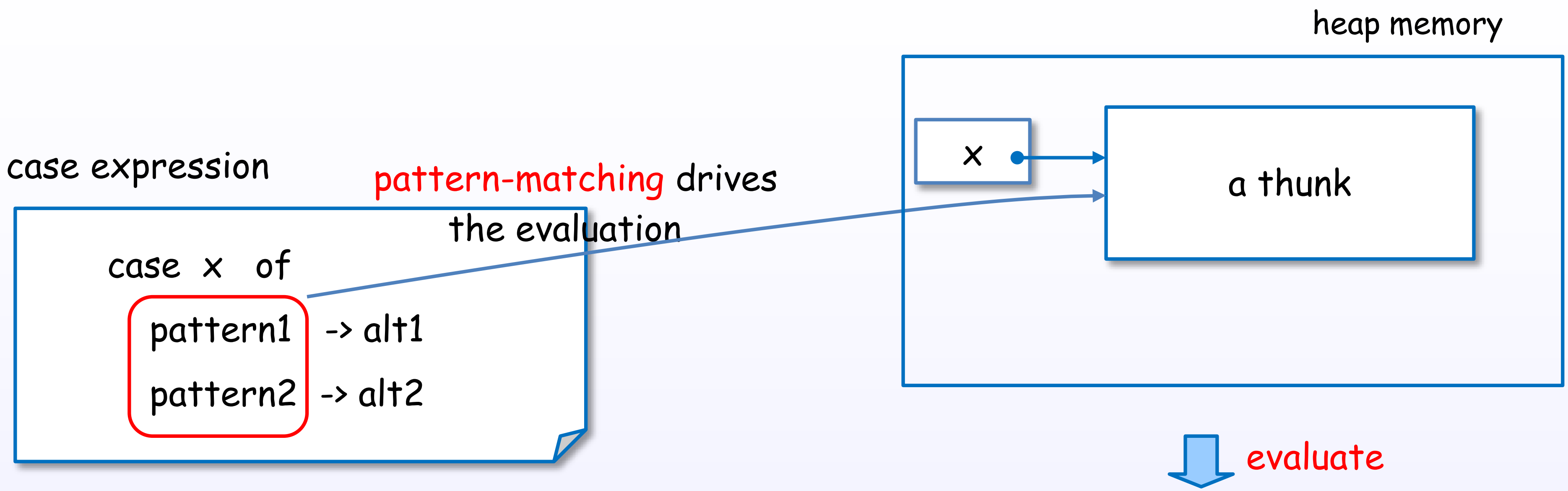
heap memory

postpone the evaluation by a thunk which build with let expression

(When GHC can optimize it by analysis, the thunk may not be build.)

References : [H5], [H6], [H10]

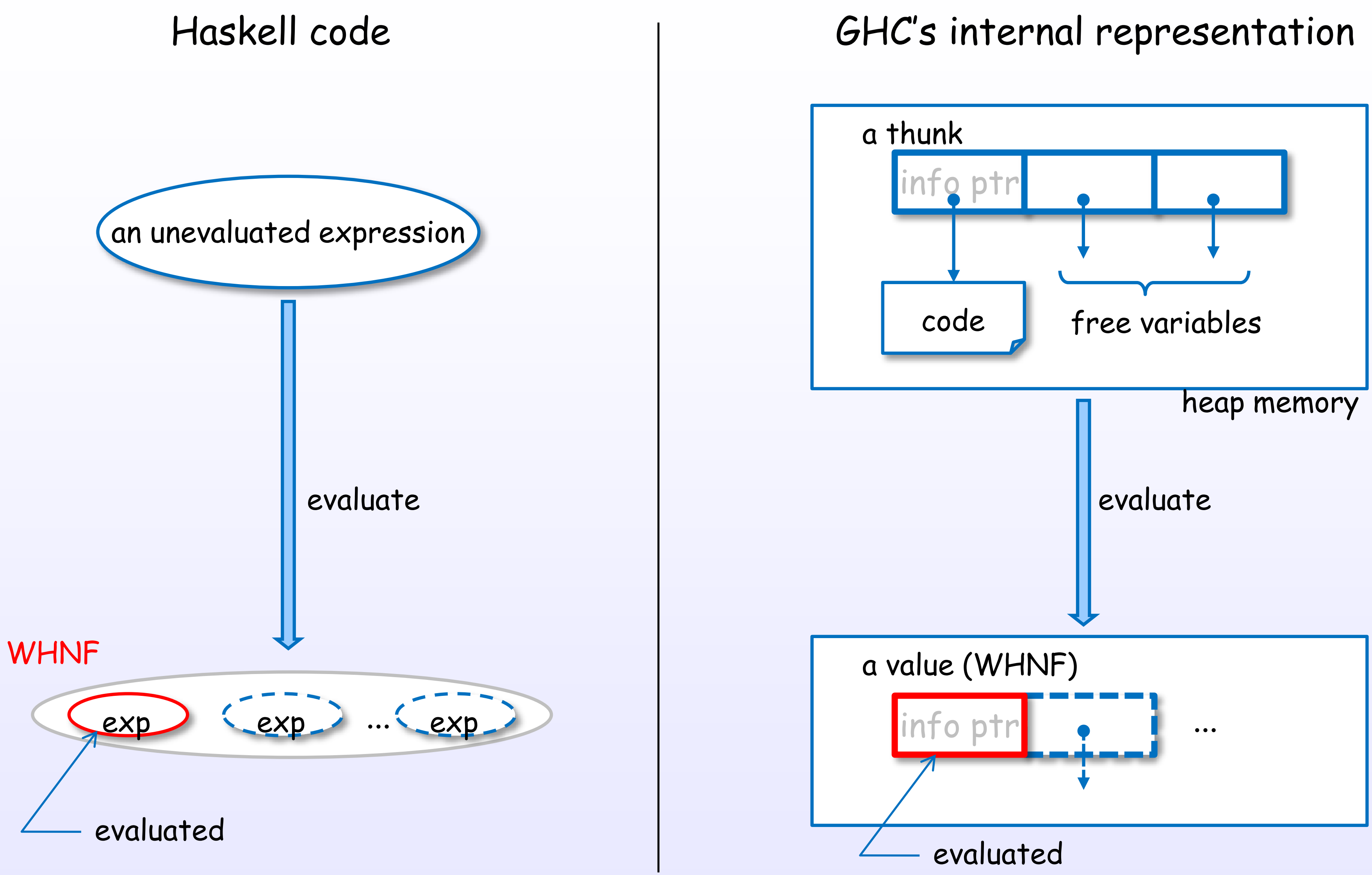
Pattern-matching drives the evaluation



drive the evaluation by pattern-matching

References : [H5], [H6], [H7], [H10], [D2]

Stop at WHNF



stop the evaluation at WHNF

References : [H5], [H6], [H10]

Examples of evaluation steps

(1) Example of GHC's evaluation

```
tail (map abs [1, -2, 3])
```

Let's evaluate. It's time to magic!

* no optimizing case (without -O)

(2) How to postpone the evaluation of arguments?

```
tail (map abs [1, -2, 3])
```

argument

function

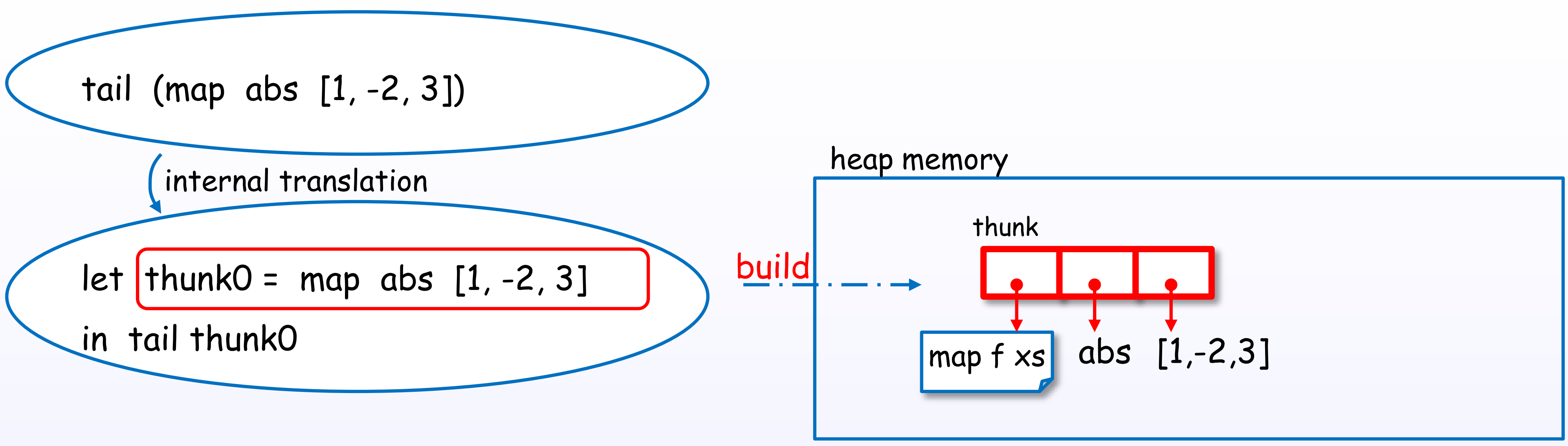
(3) GHC internally translates the expression

```
tail (map abs [1, -2, 3])
```

internal translation

```
let thunk0 = map abs [1, -2, 3]
in tail thunk0
```

(4) a let expression builds a thunk



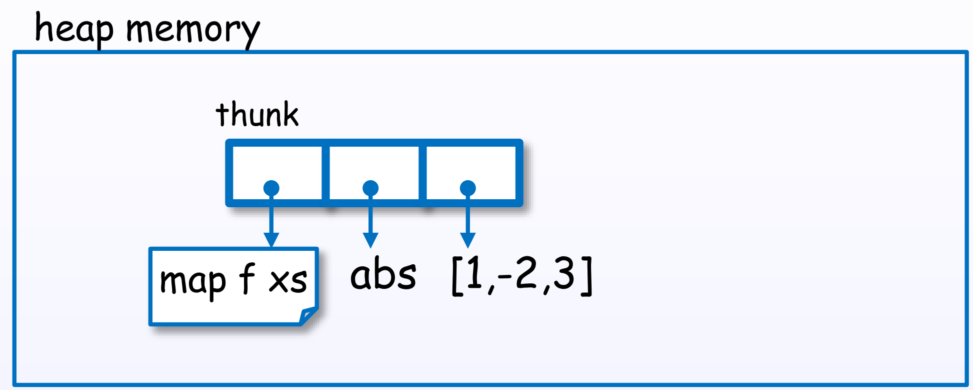
(5) function apply to argument

```
tail  (map  abs  [1, -2, 3])
```

internal translation

```
let  thunk0 =  map  abs  [1, -2, 3]
in  tail thunk0
```

apply



(6) tail function is defined here

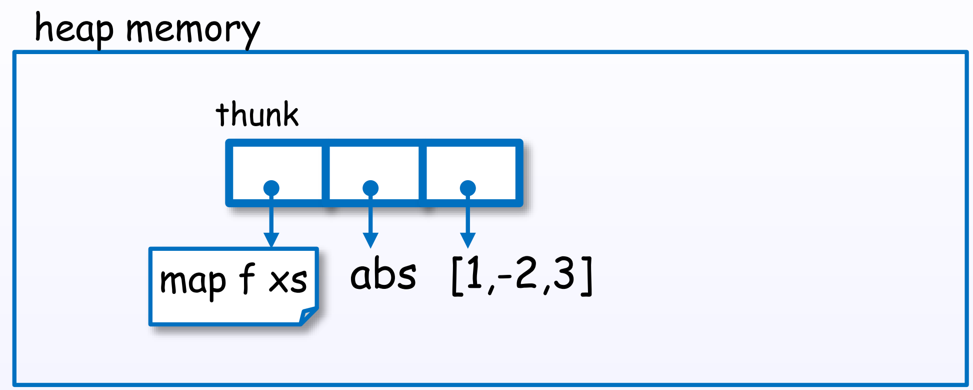
```
tail (map abs [1, -2, 3])
```

internal translation

```
let thunk0 = map abs [1, -2, 3]
in tail thunk0
```

definition

```
tail (_:xs) = xs
```



(7) function's pattern is syntactic sugar

```
tail (map abs [1, -2, 3])
```

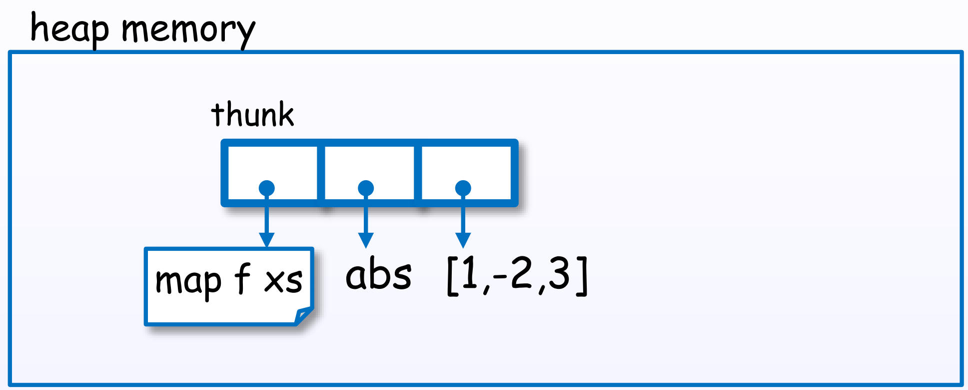
internal translation

```
let thunk0 = map abs [1, -2, 3]
in tail thunk0
```

```
tail (_:xs) = xs
```

syntactic desugar

```
tail y = case y of
           (_:xs) -> xs
```



(8) substitute the function body (beta reduction)

tail (map abs [1, -2, 3])

internal translation

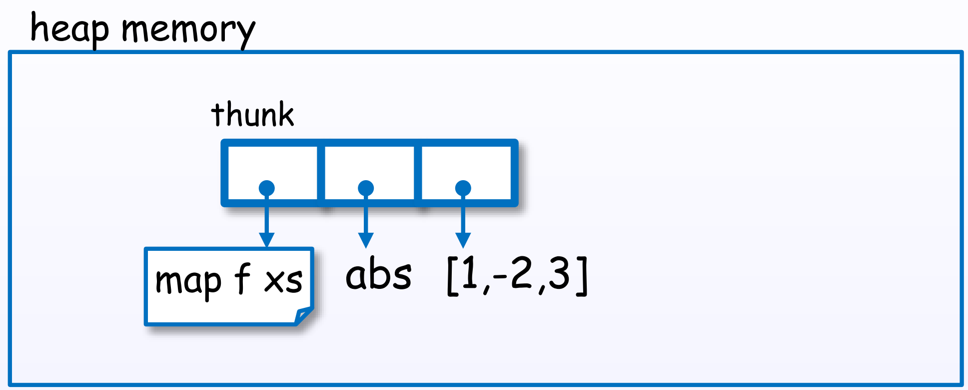
let thunk0 = map abs [1, -2, 3]
in tail thunk0

reduction

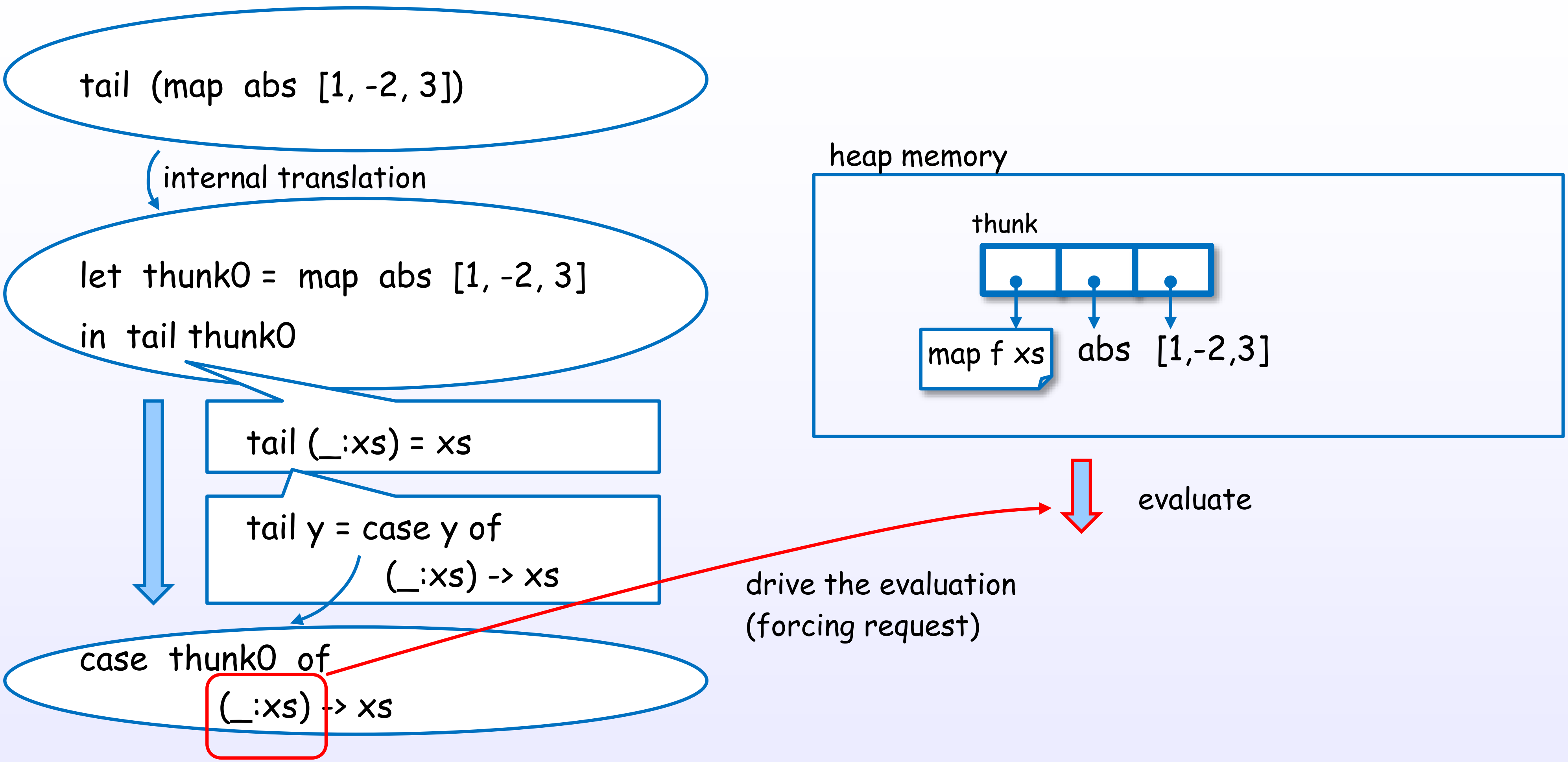
tail (_:xs) = xs

tail y = case y of
(_:xs) -> xs

case thunk0 of
(_:xs) -> xs



(9) case pattern-matching drives the evaluation



(10) but, stop at WHNF

```
tail (map abs [1, -2, 3])
```

internal translation

```
let thunk0 = map abs [1, -2, 3]
in tail thunk0
```

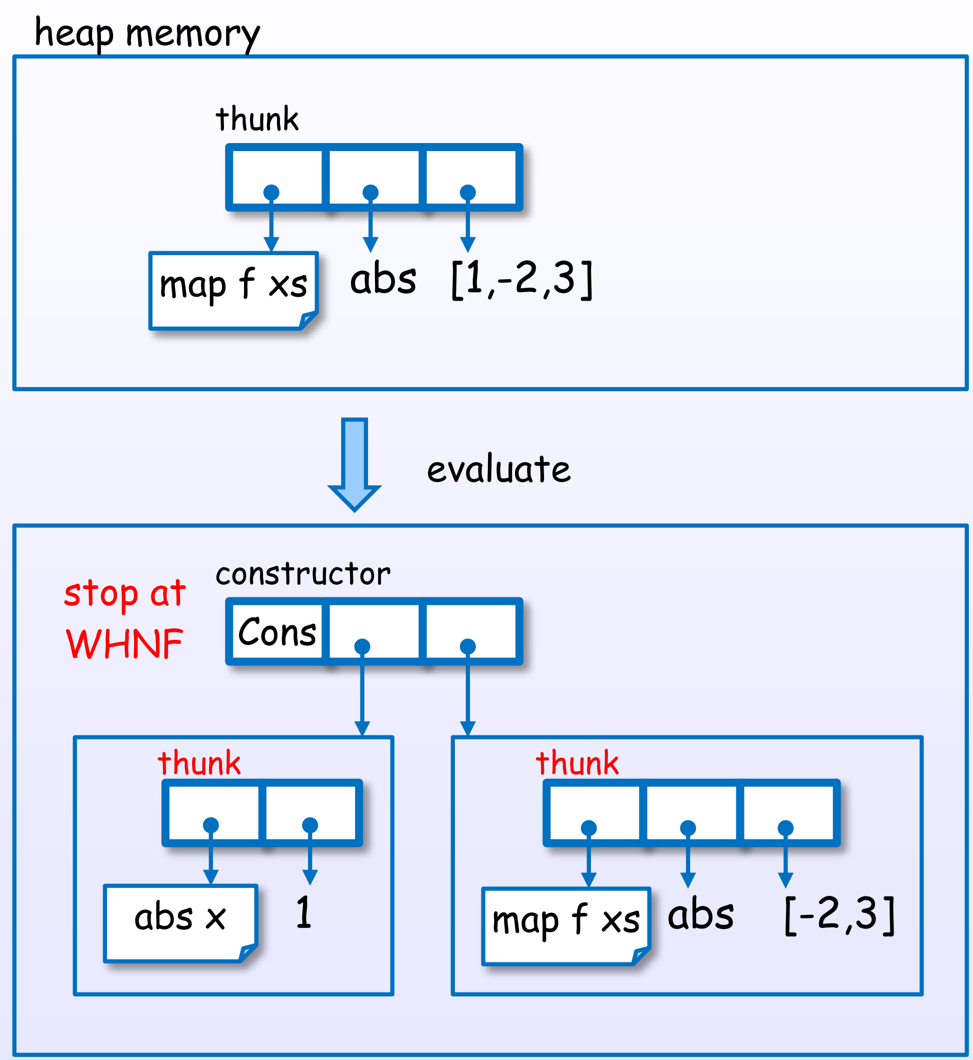
```
tail (_:xs) = xs
```

```
tail y = case y of
           (_:xs) -> xs
```

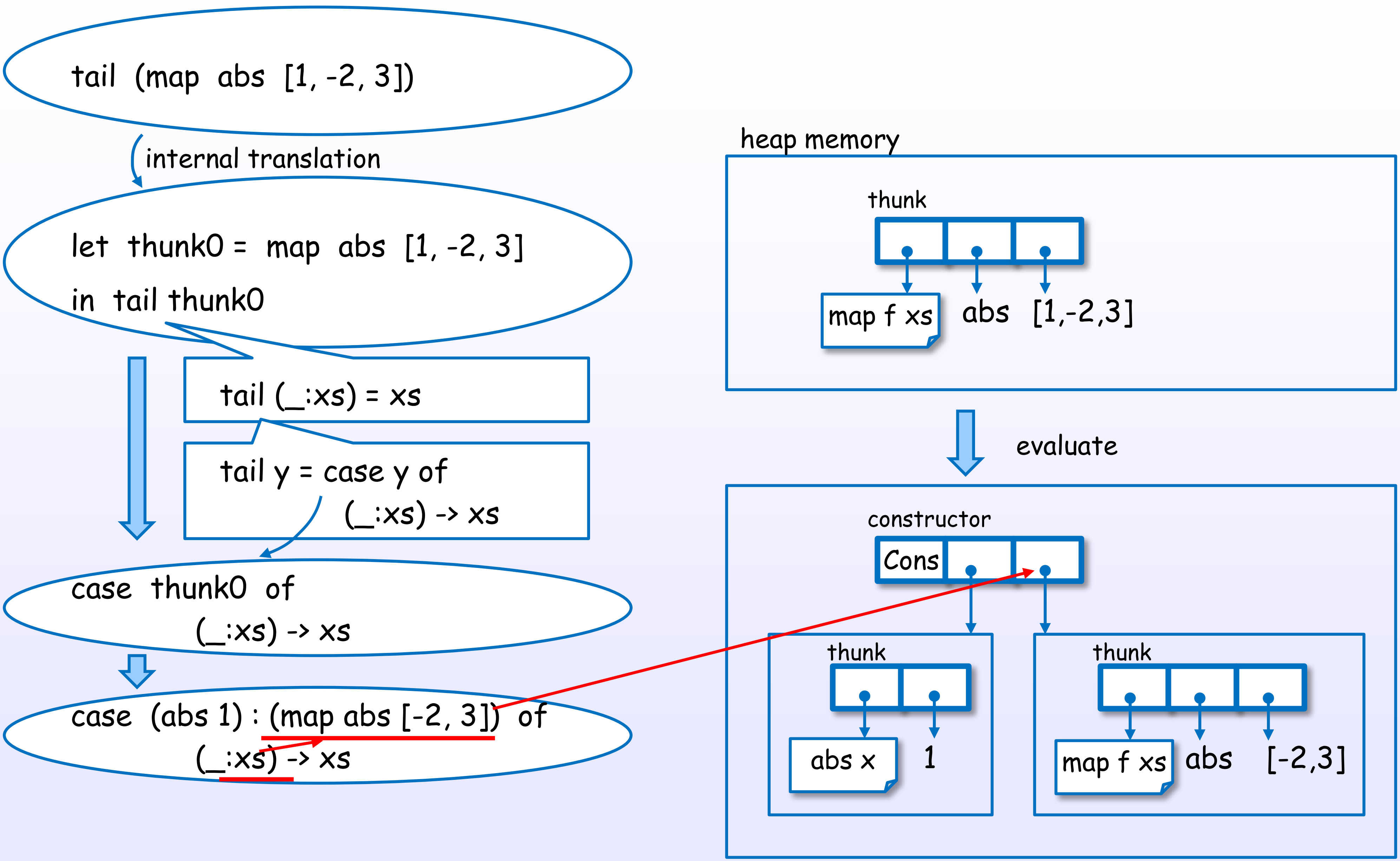
```
case thunk0 of
       (_:xs) -> xs
```

evaluated

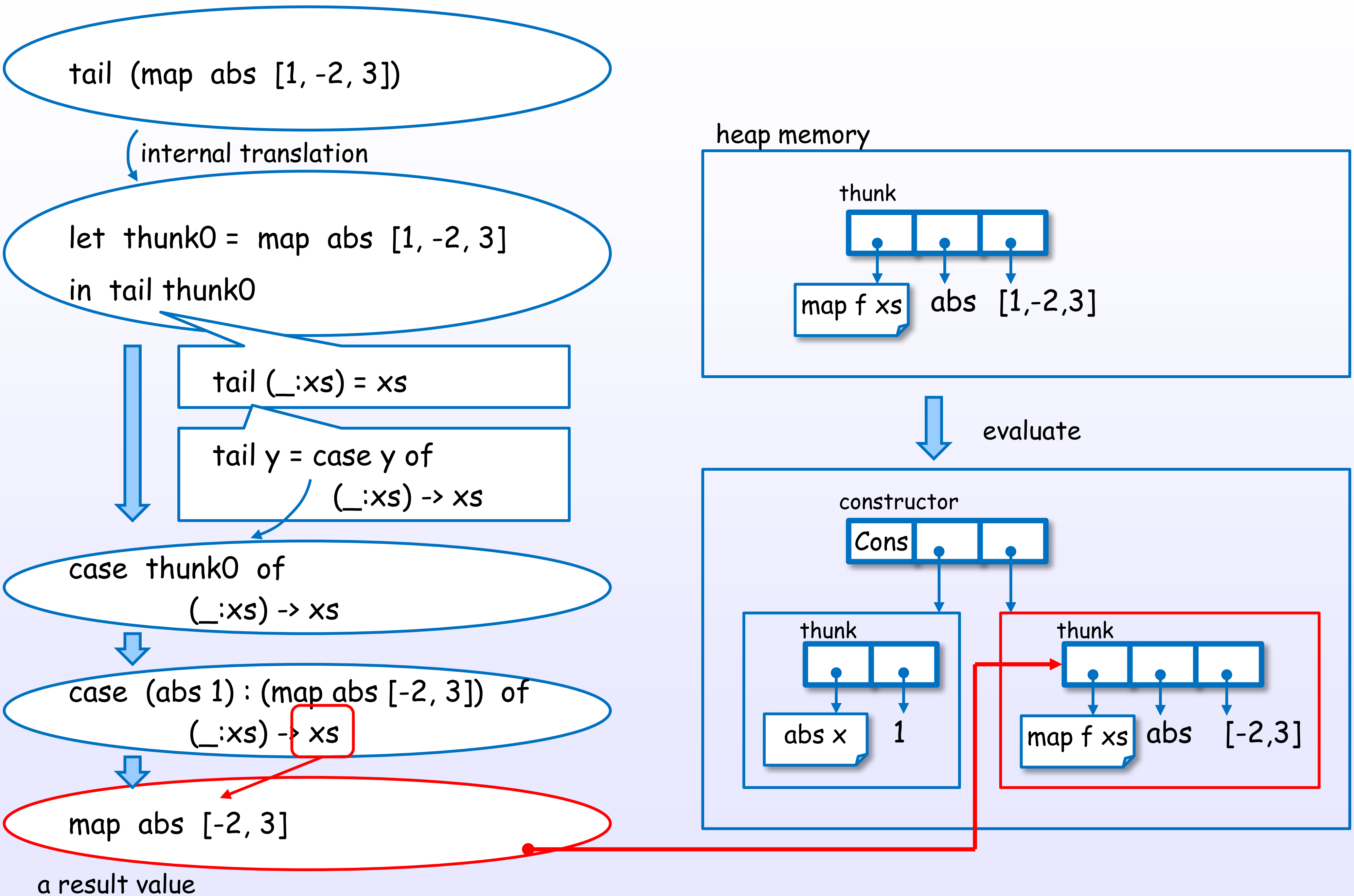
```
case (abs 1) : (map abs [-2, 3]) of
       (_:xs) -> xs
```



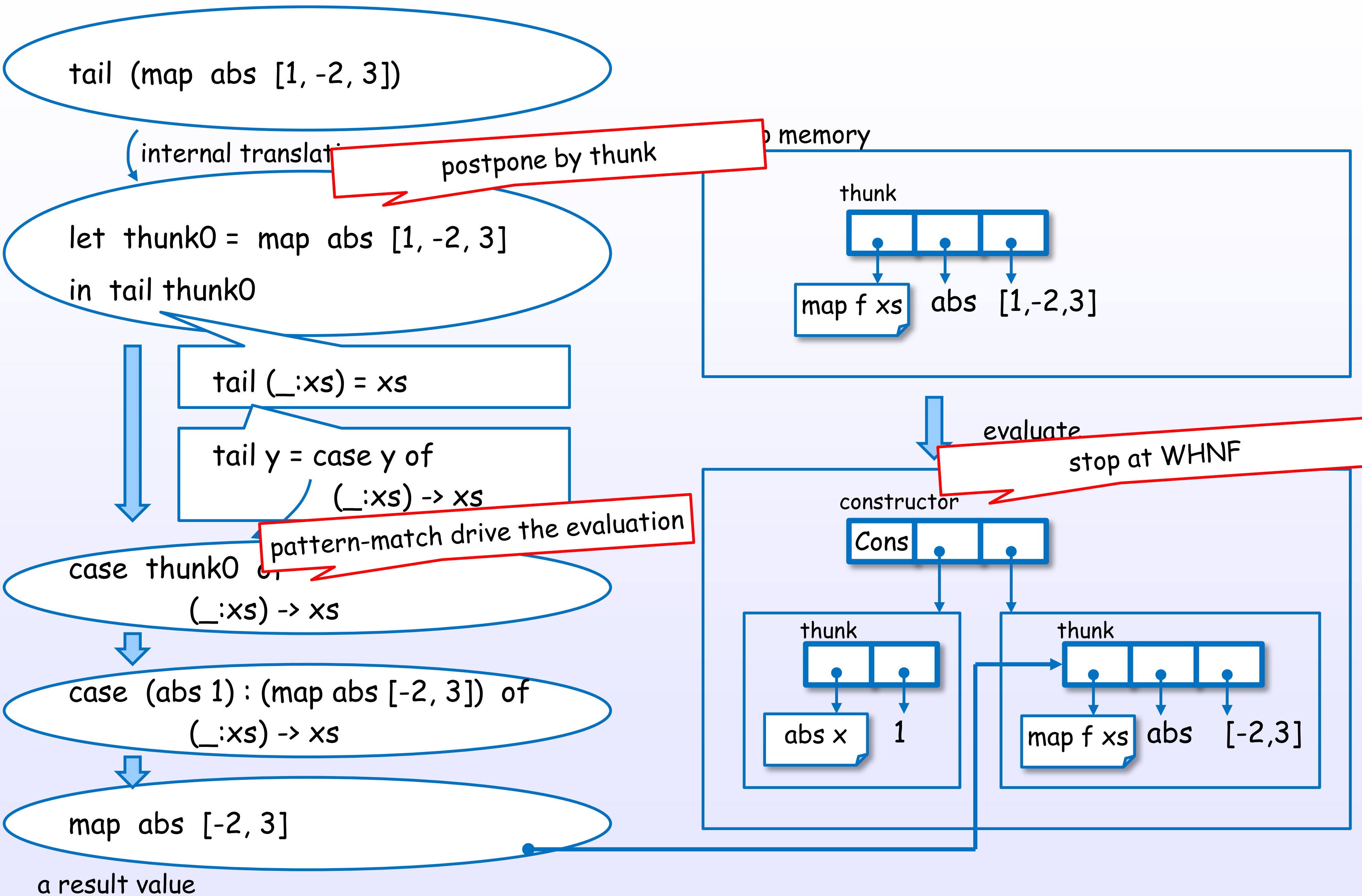
(11) bind variables to a result



(12) return the value



Key points


tail (map abs [1, -2, 3])
internal translat
postpone by thunk
memory
let thunk0 = map abs [1, -2, 3]
in tail thunk0
thunk
map f xs
abs
[1,-2,3]
tail (_:xs) = xs
tail y = case y of
(_:xs) -> xs
evaluate
stop at WHNF
constructor
Cons
pattern-match drive the evaluation
case thunk0 of
(_:xs) -> xs
case (abs 1) : (map abs [-2, 3]) of
(_:xs) -> xs
thunk
abs x
1
thunk
map f xs
abs
[-2,3]
map abs [-2, 3]
a result value

4. Evaluation

Examples of evaluations

* no optimizing case (without -O)

Example of repeat

repeat 1

1 : repeat 1

1 : 1 : repeat 1

1 : 1 : 1 : repeat 1

References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of repeat

```
repeat 1
```

```
1 : repeat 1
```

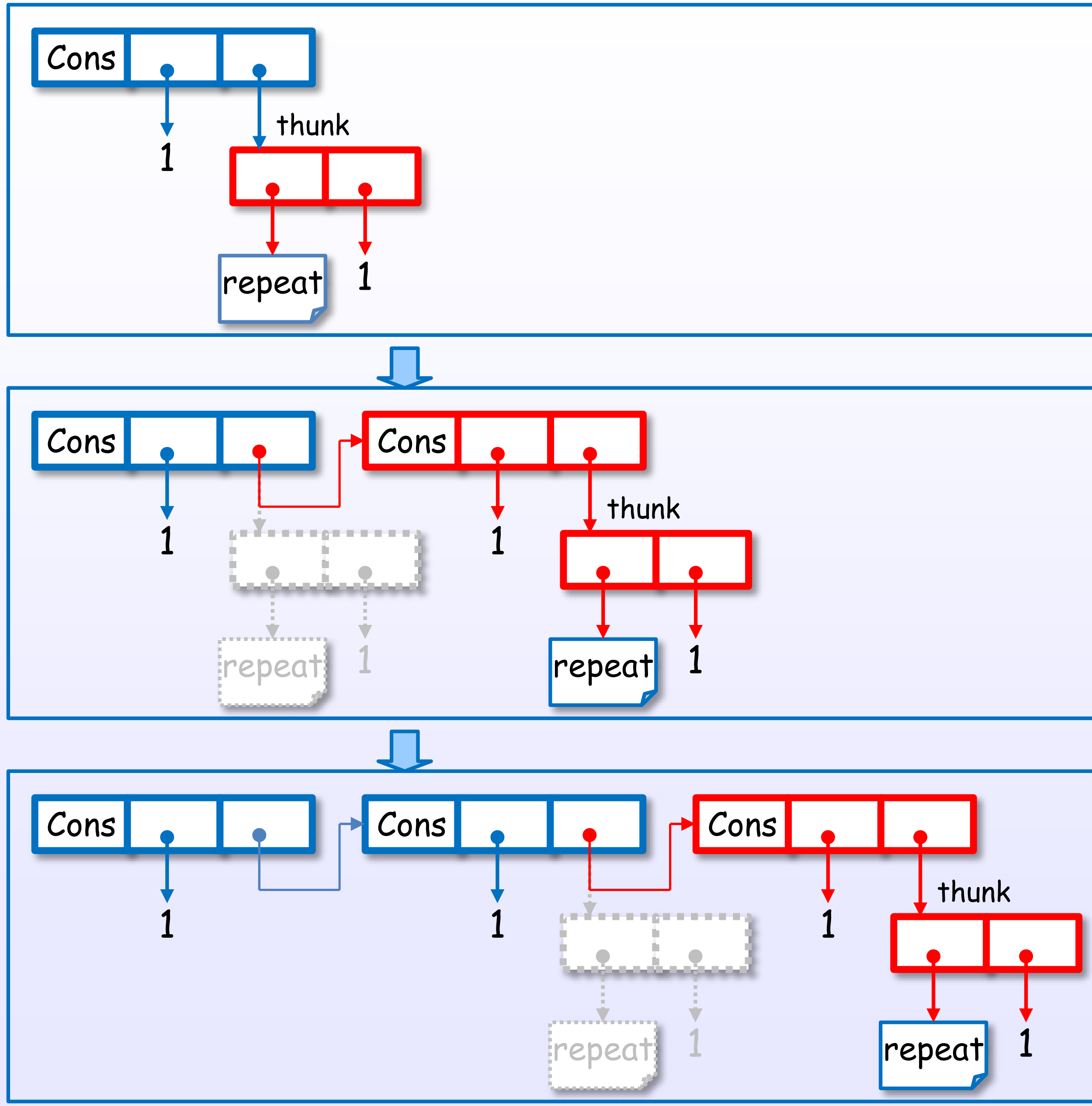
```
Cons 1 (repeat 1)
```

```
1 : 1 : repeat 1
```

```
Cons 1 (Cons 1 (repeat 1) )
```

```
1 : 1 : 1 : repeat 1
```

```
Cons 1 (Cons 1 (Cons 1 (repeat 1) ) )
```



References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of map

map f [1, 2, 3]

f 1 : map f [2, 3]

f 1 : f 2 : map f [3]

f 1 : f 2 : f 3

...

References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of map

map f [1, 2, 3]

f 1 : map f [2, 3]

Cons (f 1) (map f [2, 3])

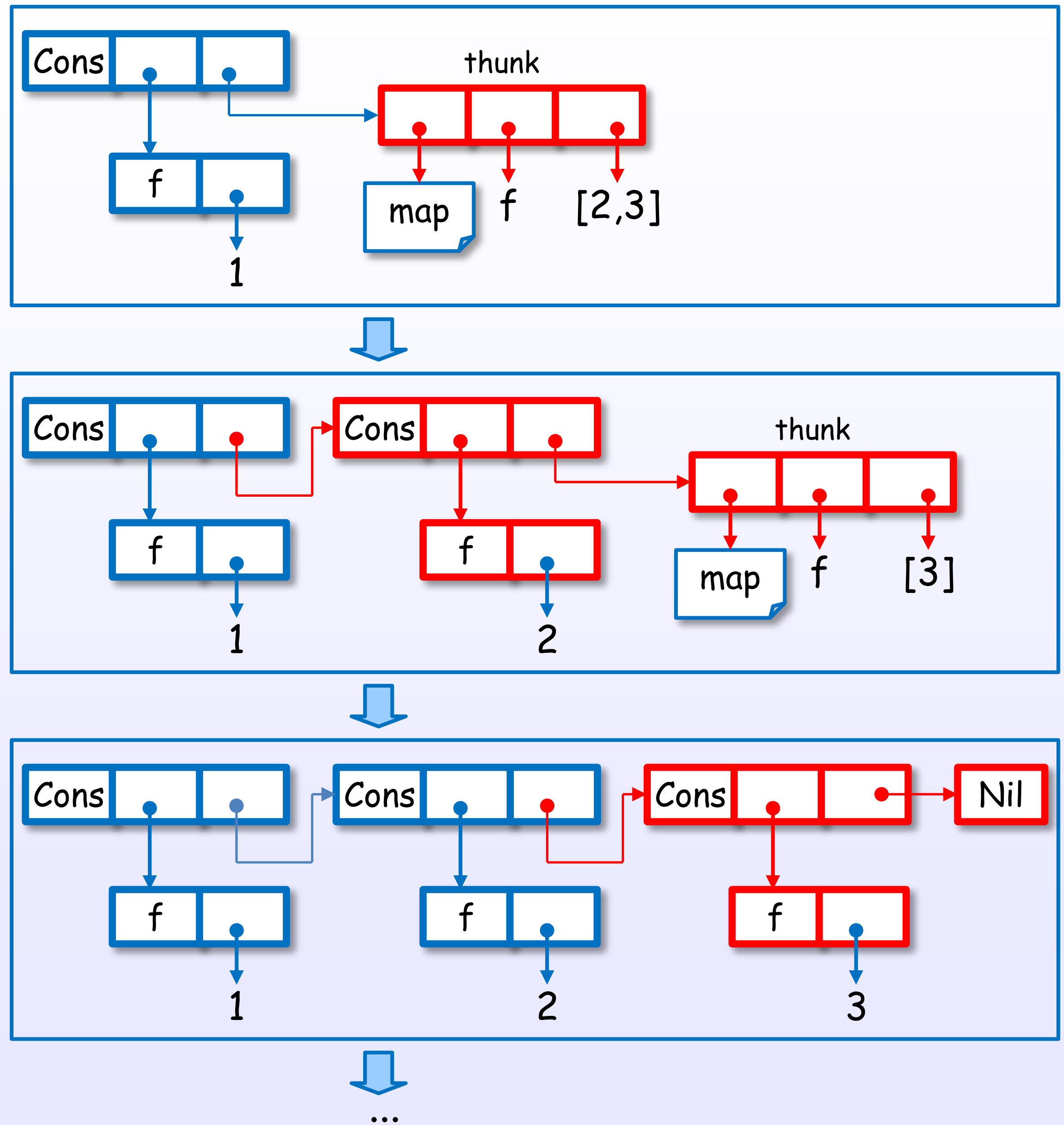
f 1 : f 2 : map f [3]

Cons (f 1) (Cons (f 2) (map f [3]))

f 1 : f 2 : f 3

Cons (f 1) (Cons (f 2) (Cons (f 3) Nil))

…



References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl (non-strict)

```
foldl (+)  0  [1 .. 100]
```

⇩

```
foldl (+) (0 + 1) [2 .. 100]
```

⇩

```
foldl  (+)  ((0 + 1) + 2)  [3 .. 100]
```

⇩

```
foldl  (+)  (((0 + 1) + 2) + 3)  [4 .. 100]
```

⇩

...

References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl (non-strict)

```
foldl (+)  0  [1 .. 100]
```

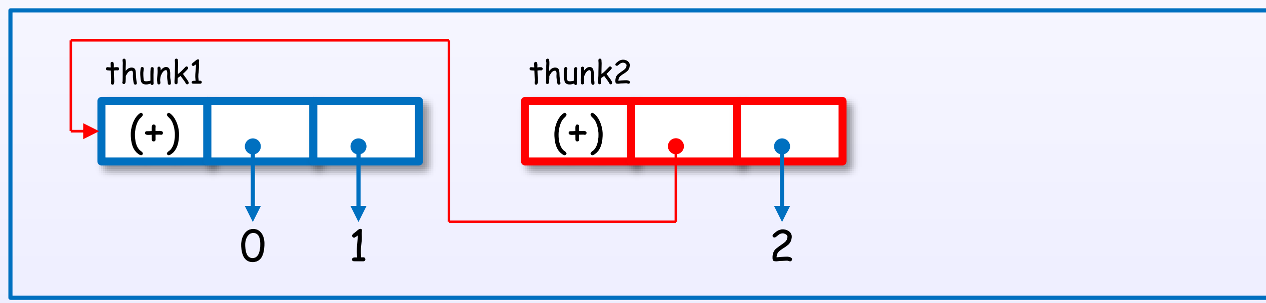
heap memory *show only accumulation value

```
foldl (+) (0 + 1) [2 .. 100]
```

```
let  thunk1 = (0 + 1)
in foldl (+) thunk1 [2 .. 100]
```

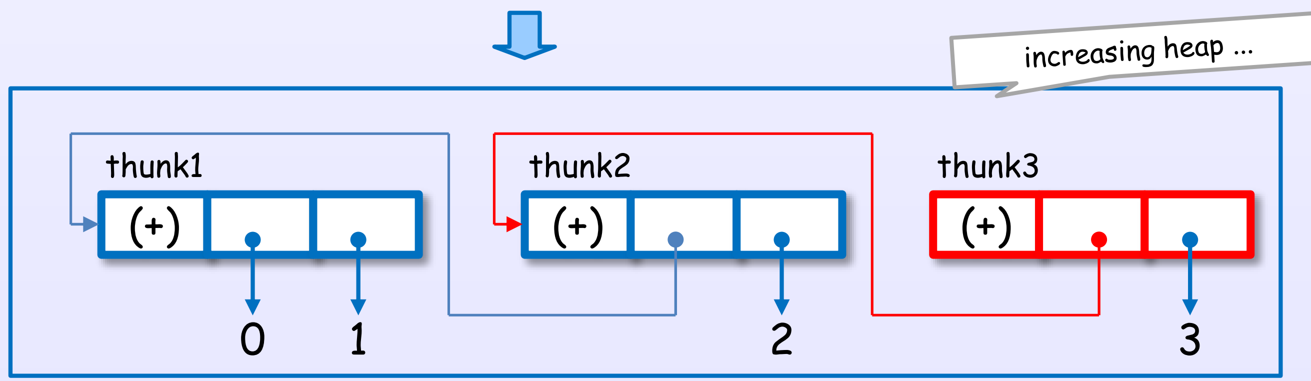

```
foldl  (+)  ((0 + 1) + 2)  [3 .. 100]
```

```
let  thunk2 = (thunk1 + 2)
in foldl (+) thunk2 [3 .. 100]
```




```
foldl  (+)  (((0 + 1) + 2) + 3)  [4 .. 100]
```

```
let  thunk3 = (thunk2 + 3)
in foldl (+) thunk3 [4 .. 100]
```



...

References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl' (strict)

foldl' (+) 0 [1 .. 100]

⇩

foldl' (+) (0 + 1) [2 .. 100]

⇩

foldl' (+) (1 + 2) [3 .. 100]

⇩

foldl' (+) (3 + 3) [4 .. 100]

⇩

...

References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl' (strict)

```
foldl' (+)  0  [1 .. 100]
```

```
foldl' (+) (0 + 1) [2 .. 100]
```

```
let  thunk1 = (0 + 1)
in  thunk1  `seq`
     foldl' (+) thunk1 [2 .. 100]
```

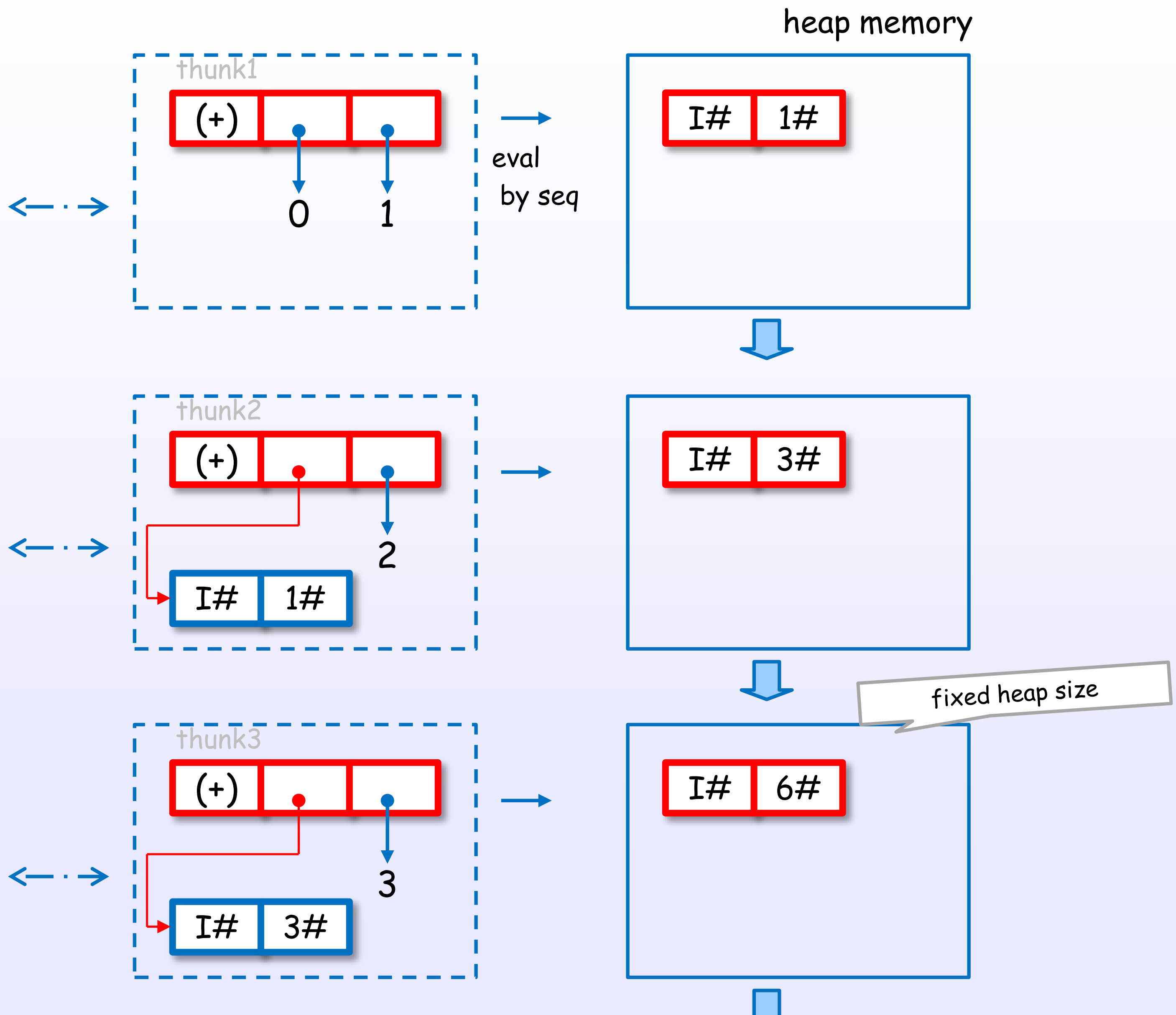
```
foldl' (+) (1 + 2) [3 .. 100]
```

```
let  thunk2 = (1 + 2)
in  thunk2  `seq`
     foldl' (+) thunk2 [3 .. 100]
```

```
foldl' (+) (3 + 3) [4 .. 100]
```

```
let  thunk3 = (3 + 3)
in  thunk3  `seq`
     foldl' (+) thunk3 [4 .. 100]
```

…



References : [D5], [D6], [D8], [D9], [D10], [H10]

Example of foldl (non-strict) and foldl' (strict)

foldl (+) (0 + 1) [2 .. 100]

foldl (+) ((0 + 1) + 2) [3 .. 100]

foldl (+) (((0 + 1) + 2) + 3) [4 .. 100]

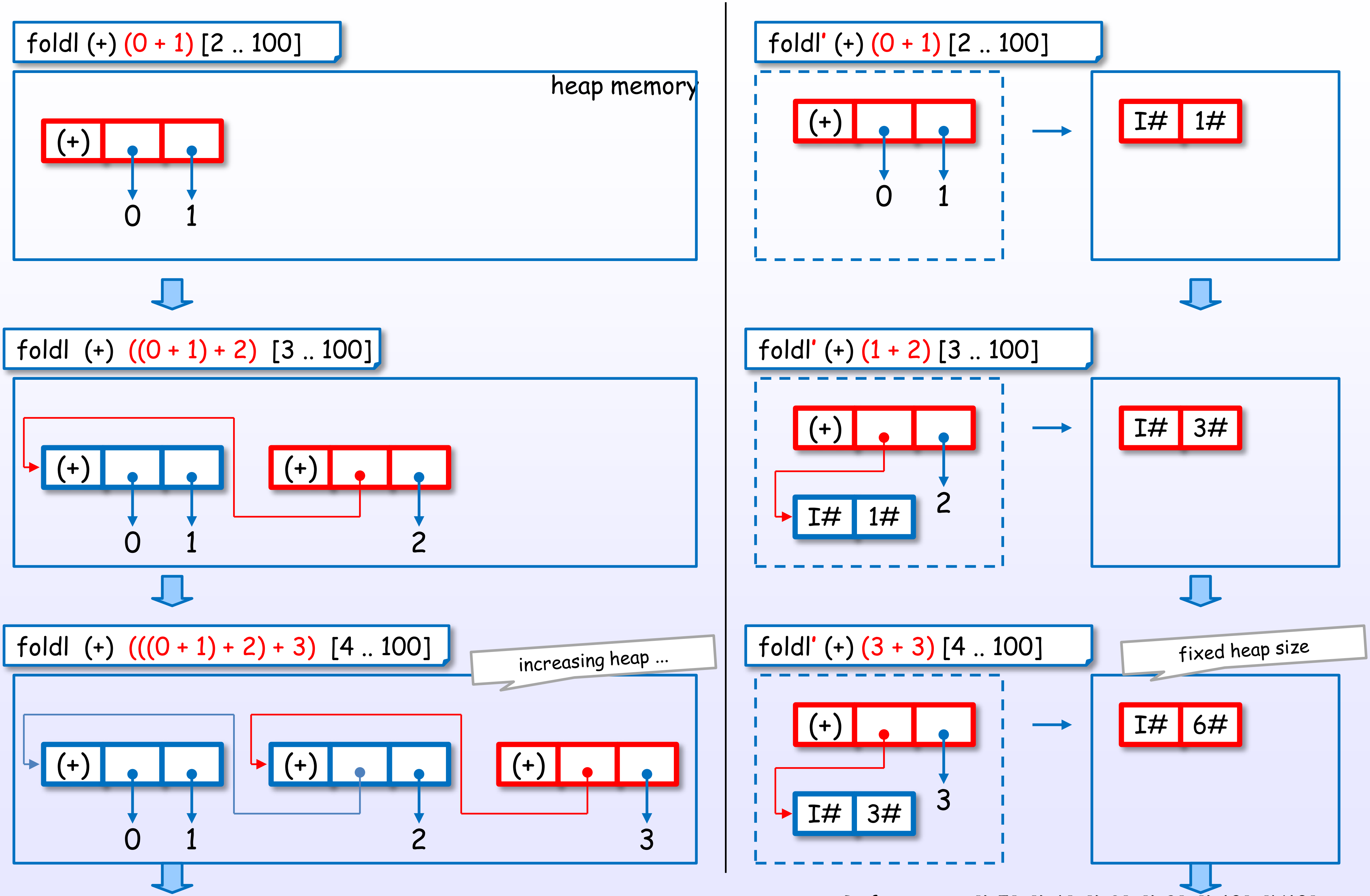
foldl' (+) (0 + 1) [2 .. 100]

foldl' (+) (1 + 2) [3 .. 100]

foldl' (+) (3 + 3) [4 .. 100]

References : [D5], [D6], [D8], [D9], [D10], [H10]

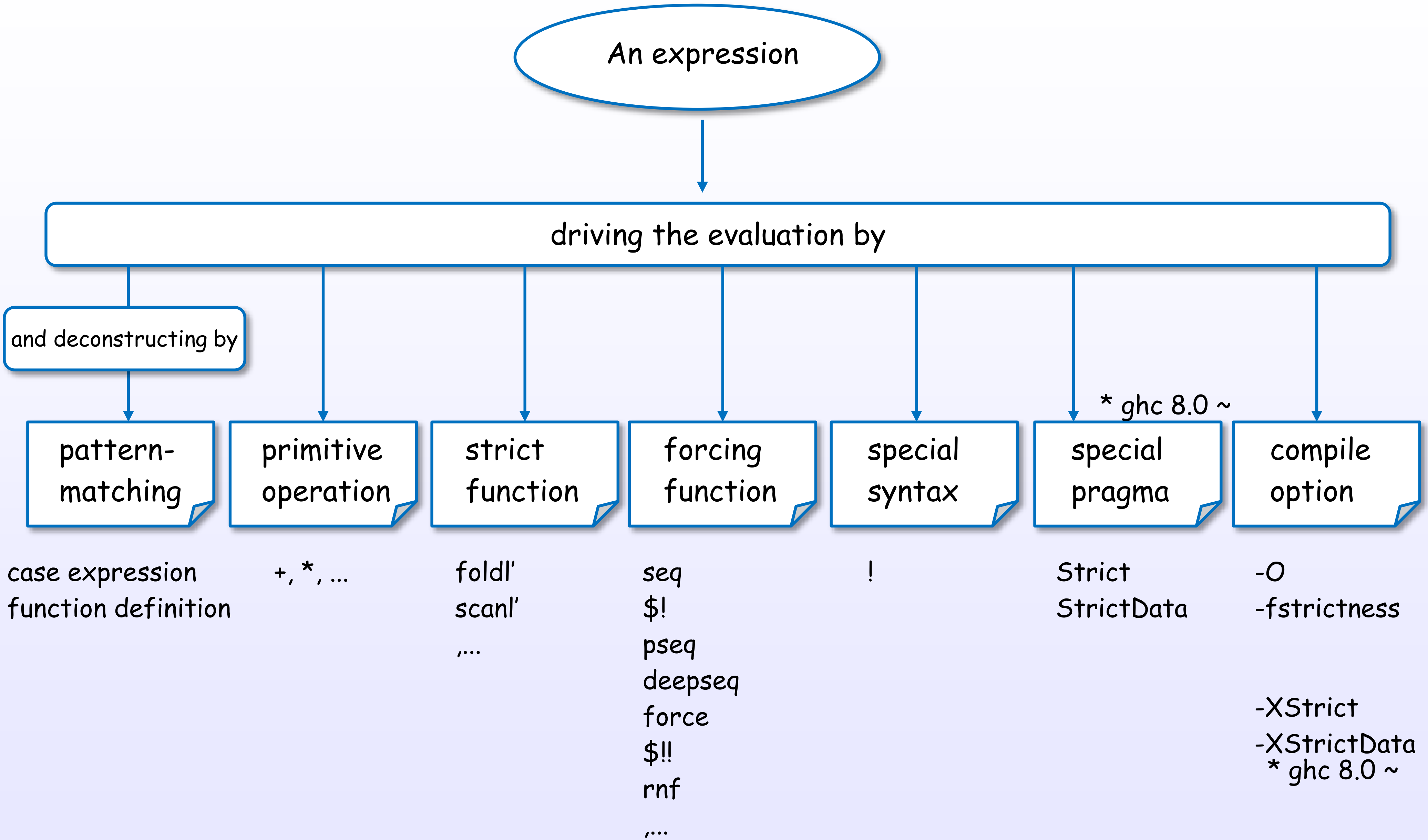
Example of foldl (non-strict) and foldl' (strict)



References : [D5], [D6], [D8], [D9], [D10], [H10]

Controlling the evaluation

How to drive the evaluation



(1) Evaluation by pattern-matching

pattern-matching in case expression

```
case ds of
  x:xs -> f x xs
  []   -> False
```

forcing
(drive the evaluation of the thunk)

pattern-matching in function definition

```
f Just _  = True
f Nothing = False
```

forcing
(drive the evaluation of the thunk)

References : [H1] Ch.3, [D2], [D1], [H5], [W1]

(1) Evaluation by pattern-matching

Strict patterns drive the evaluation

case expression

```
case  ds  of
   x:xs  -> f  x  xs
   []    -> False
```

function definition

```
f  Just _    =  True
f  Nothing   =  False
```

Lazy patterns postpone the evaluation.

let binding pattern

```
let  (x:xs)  =  fun  args
```

irrefutable patterns [H1] 3.17

```
f  ~(Just _ )  =  True
f  ~(Nothing)  =  False
```

There are two kinds of pattern-matching.

References : [H1] Ch.3, [D2], [D1], [H5], [W1], [H8] Ch.4

(2) Evaluation by primitive operation

primitive (built-in) operation

```
f x y = x + y
```

forcing x and y
(drive the evaluation of the thunks)

+, *, ...

primitive operations are defined such as

* *pseudo code*

```
(+) (I# a) (I# b) = I# (a+b)
```

pattern-matching

References : [D3], [H5], [H12]

(3) Evaluation by strict version function

strict version function

strict application of the operator

```
foldl' (+) 0 xs
```

```
scanl' (+) 0 xs
```

⋮

References : [B4] Ch.25, [D9], [B6] Ch.7, [B2], [S1], [S2]

(4) Evaluation by forcing function

forcing functions to WHNF

```
seq x y
```

forcing
(drive the evaluation of the thunk)

```
f $! x
```

```
pseq x y
```

forcing functions to NF

```
deepseq x y
```

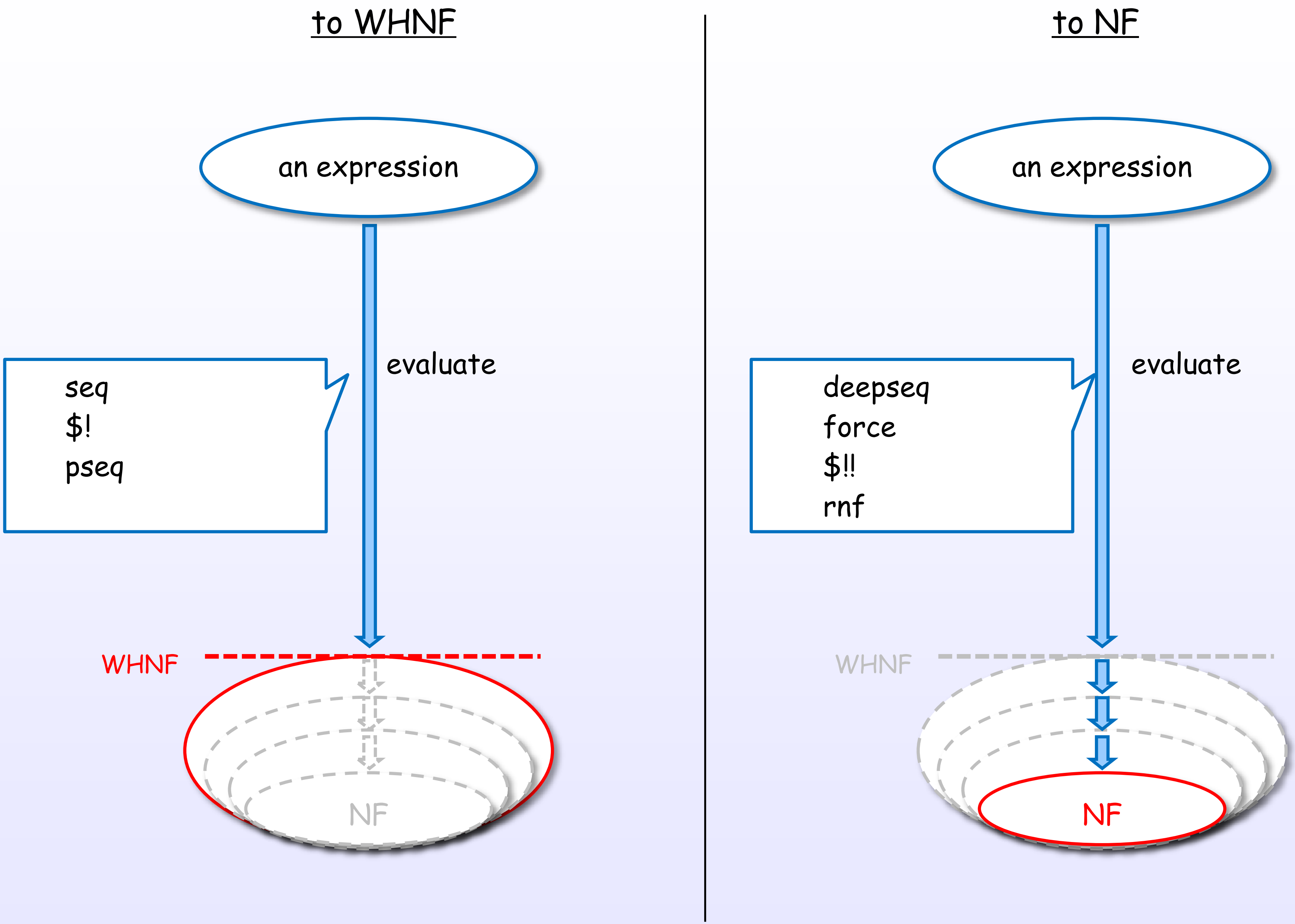
```
f $!! x
```

```
force x
```

```
rnf x
```

References : [B5] Ch.2, [B4] Ch.24, 25, [B6] Ch.7, [H1] Ch.6, [D2], [B2], [D1], [D2]. [S1], [S2]

(4) Evaluation by forcing function



References : [B5] Ch.2, [B4] Ch.24, 25, [H1] Ch.6, [D2], [B2], [D1], [D2]. [S1], [S2]

(4) Evaluation by forcing function

	to WHNF	to NF
two arguments	seq	deepseq
one argument		force
function application	$!	$!!
sequential order	pseq	
basic operation		rnf

References : [B5] Ch.2, [B4] Ch.24, 25, [H1] Ch.6, [D2], [B2], [D1], [D2]. [S1], [S2]

(4) Evaluation by forcing function

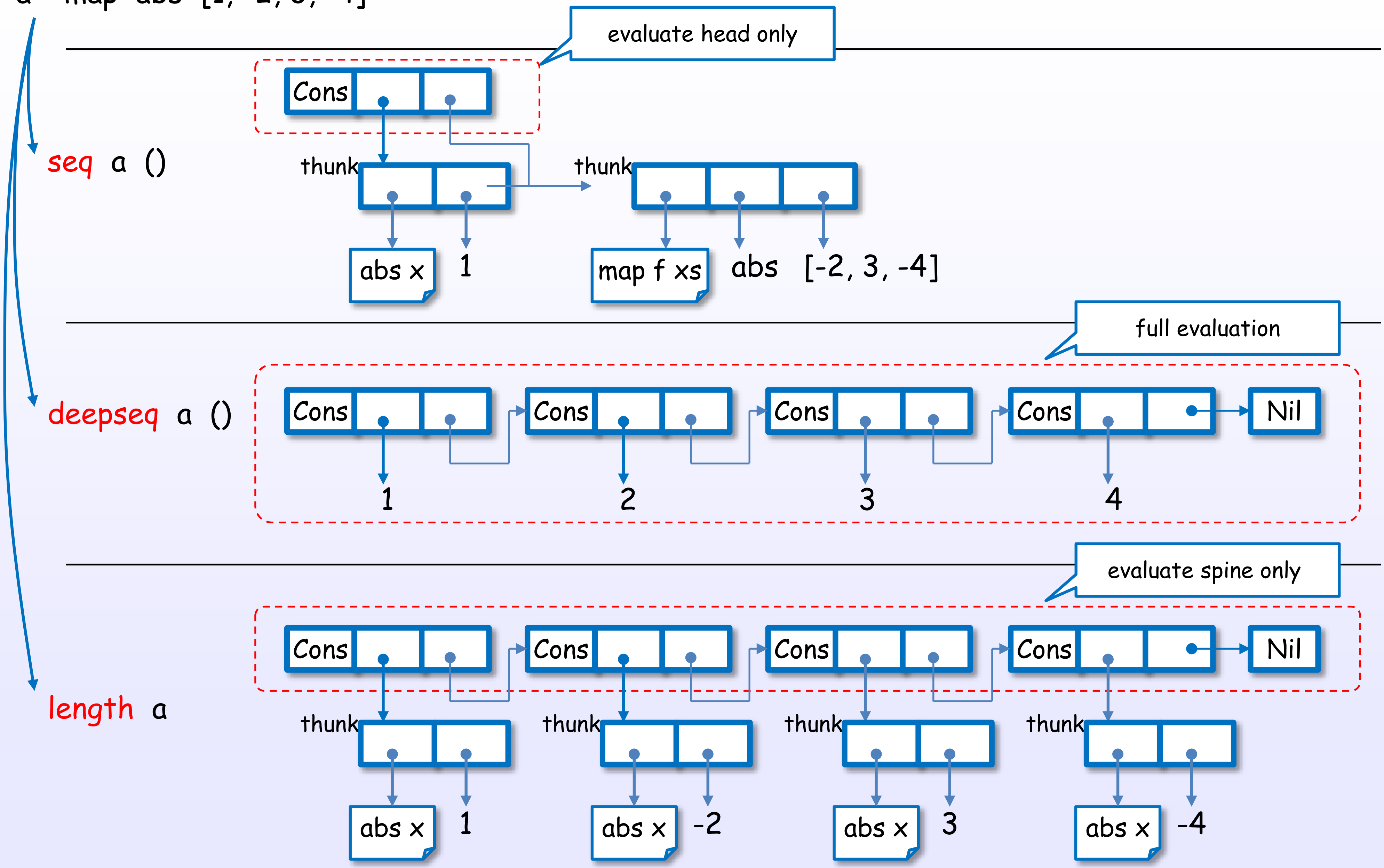
	to WHNF	to NF
two expressions	seq built-in	deepseq definition deepseq a b = rnf a `seq` b
one expression		force force x = x `deepseq` x
function application	$! f $! x = let !vx = x in f vx	$!! f $!! x = x `deepseq` f x
sequential order	pseq pseq x y = x `seq` lazy y	
basic operation		rnf

```
class NFData a where
   rnf :: a -> ()

instance NFData Int where rnf !_ = ()
   :
instance NFData a => NFData [a] where
   rnf [] = ()
   rnf (x:xs) = rnf x `seq` rnf xs
```

References : [B5] Ch.2, [B4] Ch.24, 25, [H1] Ch.6, [D2], [B2], [D1], [D2]. [S1], [S2]

(4) Evaluation by forcing function

a = map abs [1, -2, 3, -4]



References : [B5] Ch.2, [B4] Ch.24, 25, [H1] Ch.6, [D2], [B2], [D1], [D10]. [S1], [S2]

(5) Evaluation by special syntax

Strictness annotation

Bang pattern [H2] 7.19

see also Strict pragma

```
{-# LANGUAGE BangPatterns #-}

f !xs = g xs
```

arguments are evaluated before function application

Strictness flag [H1] 4.2.1

see also StrictData and Strict pragma

```
data Pair = Pair !a !b
```

arguments are evaluated before constructor application

Strictness annotations assist strictness analysis.

References : [D1], [H2] Ch.7, [H1] Ch.4, [B4] Ch.25, [B2] Ch.7, [W6], [H4] Ch.22, [W4]

(6) Evaluation by special pragma

Special pragma for strictness language extension

Strict pragma * ghc 8.0 ~

see also bang pattern and strictness flag

```
{-# LANGUAGE Strict #-}

let f xs = g xs in f ys
data Pair = Pair a b
```

arguments are evaluated before application

StrictData pragma * ghc 8.0 ~

see also strictness flag

```
{-# LANGUAGE StrictData #-}

data Pair = Pair a b
```

Strict and StrictData pragmas are module level control.
These can use in ghc 8.0 or later.

References : [H13]

(7) Evaluation by compile option

Compile option

strictness analysis

```
$ ghc -O
```

Turn on optimization.
Imply "-fstrictness".

```
$ ghc -fstrictness
```

Turn on strictness analysis.
Implied by "-O".

strictness language extension

* ghc 8.0 ~

```
$ ghc -XStrict
```

apply Strict pragma

```
$ ghc -XStrictData
```

apply StrictData pragma

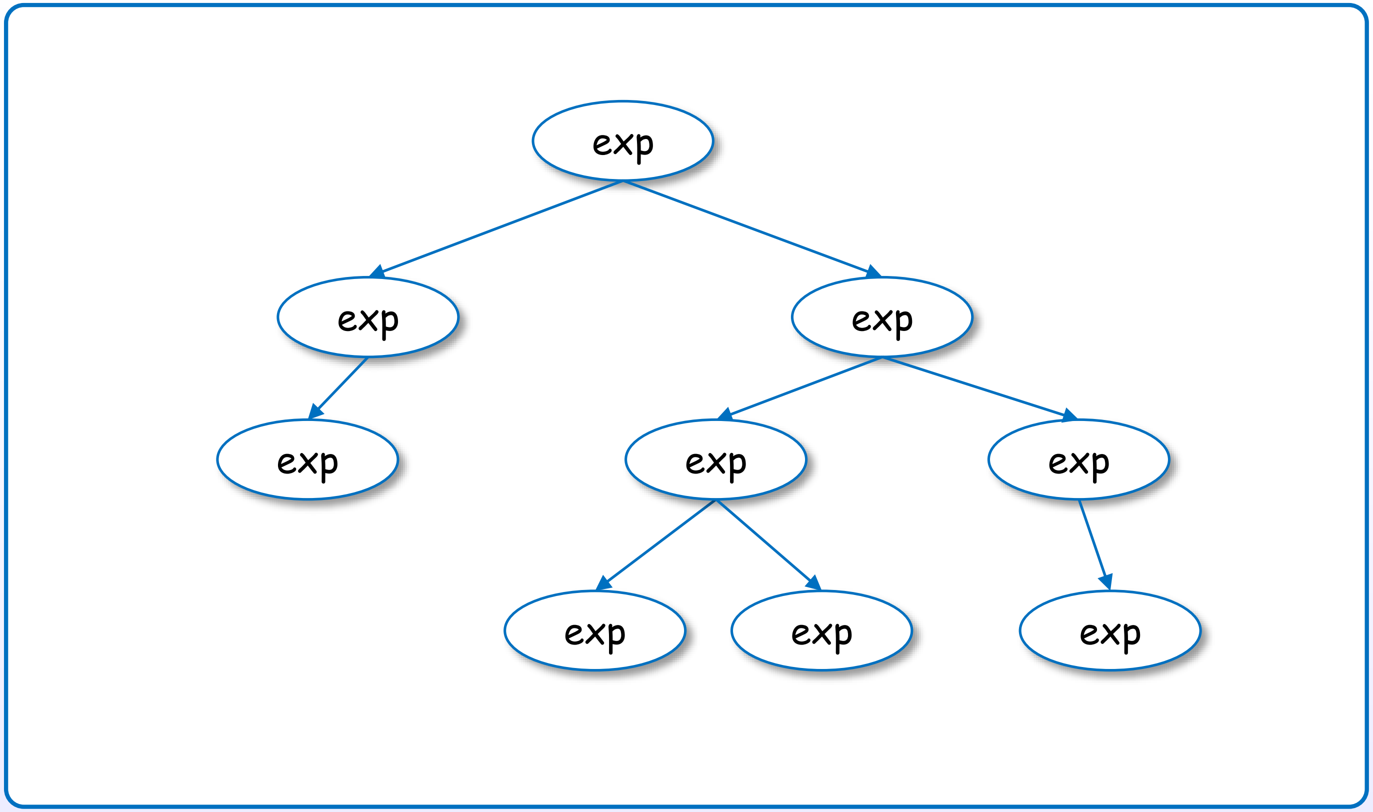
References : [H2], [H13]

5. Implementation of evaluator

5. Implementation of evaluator

Lazy graph reduction

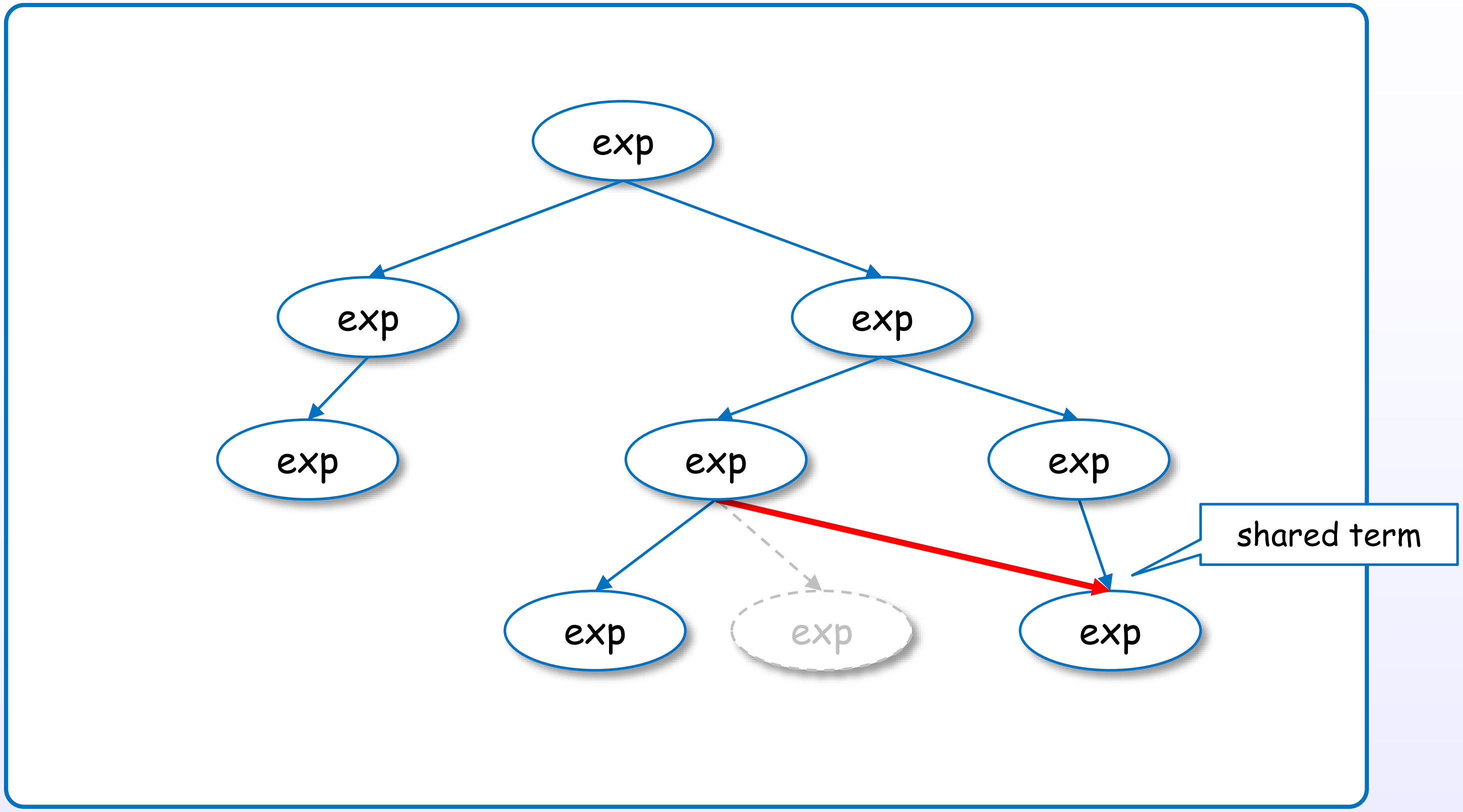
Tree



An expression can be represented in the form of Abstract Syntax Tree (AST).
AST is reduced using stack (sequential access memory).

References : [D3], [D2], [D5], [W5], [H4] Ch.12, [B8] Ch.3

Graph



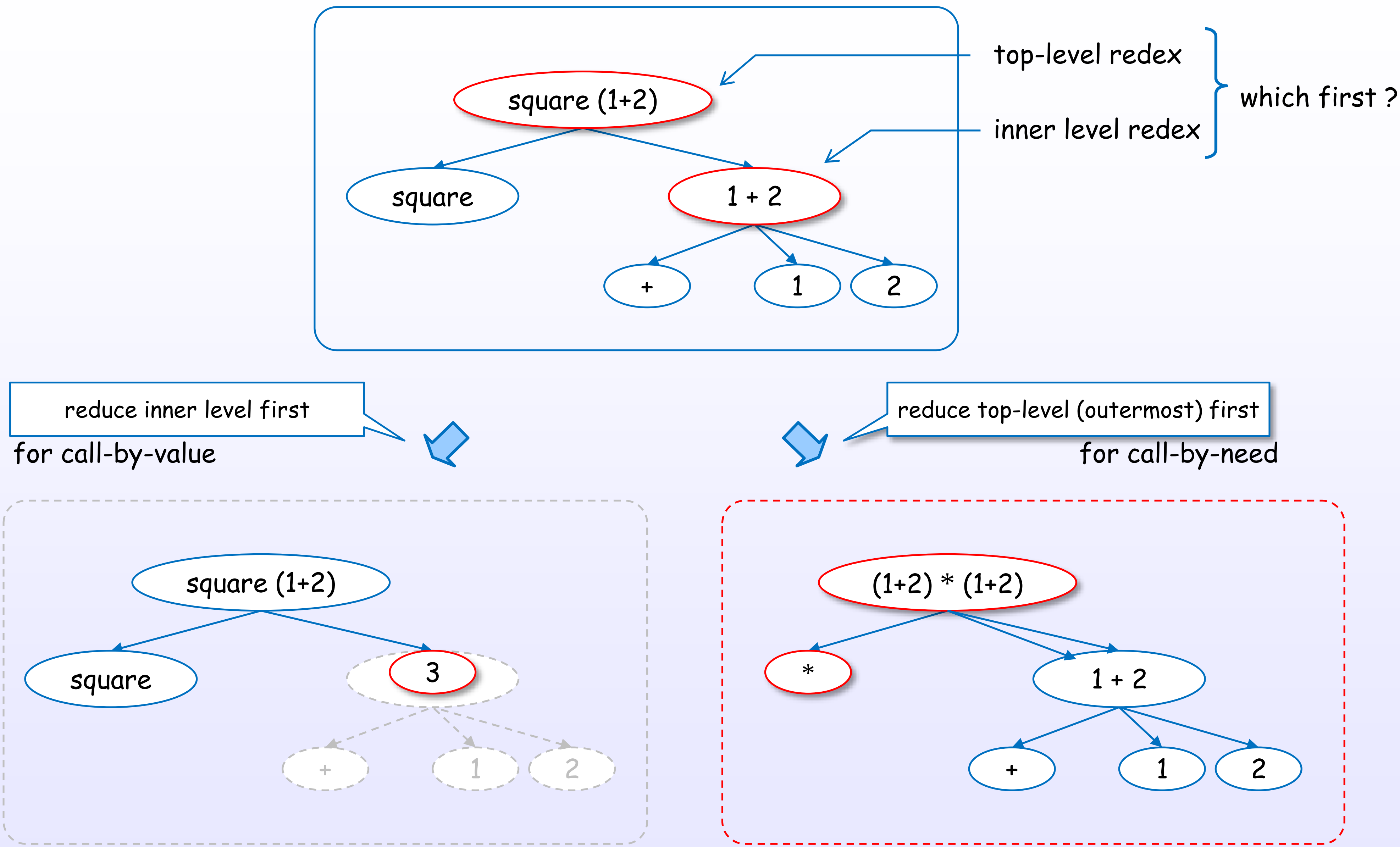
An expression can be also represented in the form of Graph.

Graph can share subexpressions to evaluate at once.

So, graph is reduced using heap (random access memory) rather than stack.

References : [D3], [D2], [D5], [W5], [H4] Ch.12, [B8] Ch.3

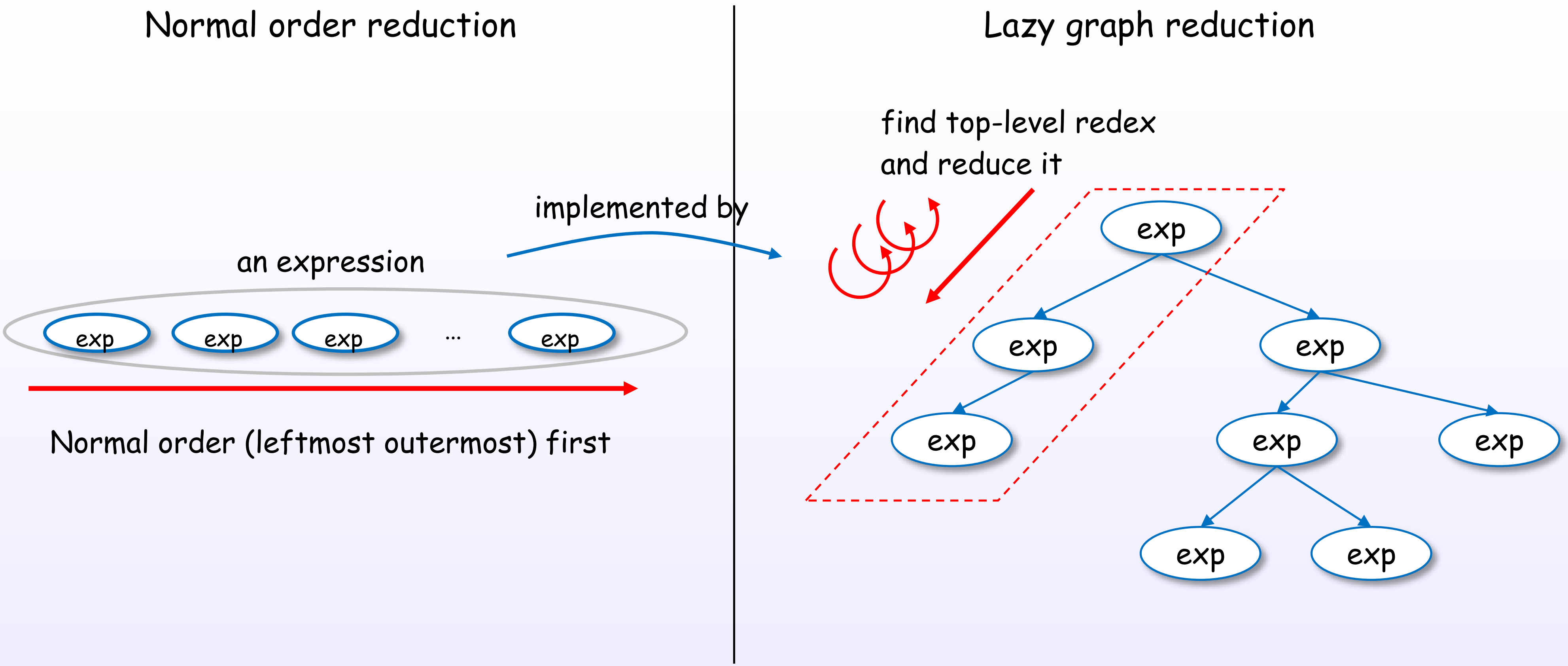
Graph can be reduced in some order



To select top-level redex first, the evaluation of arguments can be postponed.

References : [D3], [W5], [H4] Ch.11, 12, [B8] Ch.3

Normal order reduction is implemented by lazy graph reduction



Normal order (leftmost outermost) reduction is implemented by lazy graph reduction to select top-level redex first.

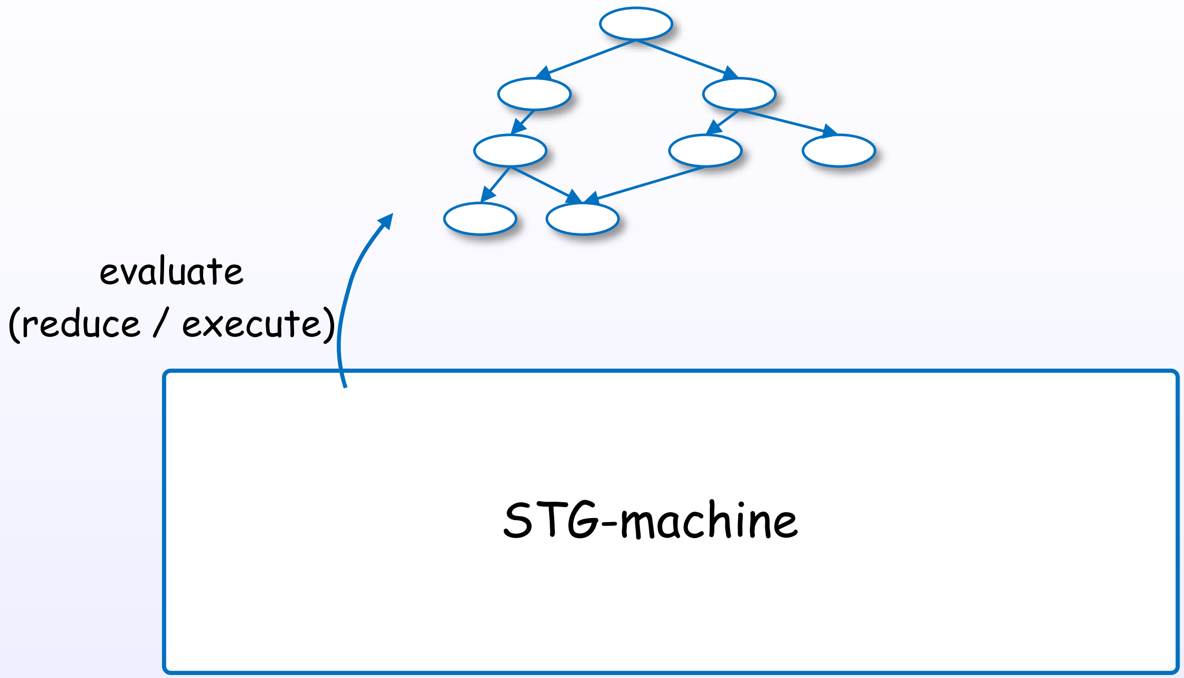
Given an application of a function, the outermost redex is the function application itself.

References : [D3], [D2], [D5], [W5], [H4] Ch.11, 12, [B8] Ch.3

STG-machine

Abstract machine

Graph
(expression)



Evaluator
(abstract machine)

GHC uses abstract machine to reduce the expression.
It's called "STG-machine".

References : [H5], [H6], [H7], [D15]

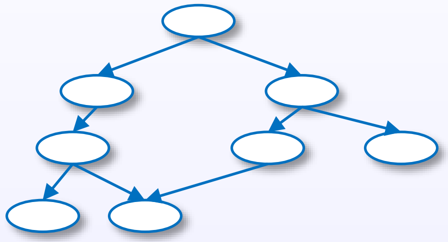
Concept layer

Haskell code

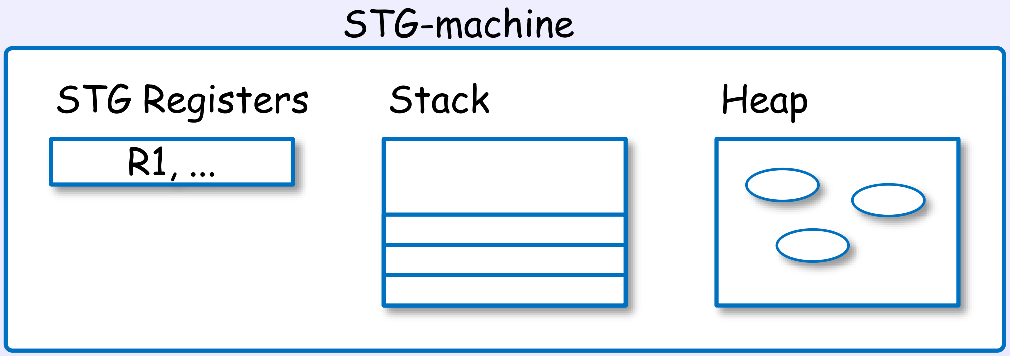
take 5 [1..10]

:

Graph
(internal representation
of the expression)

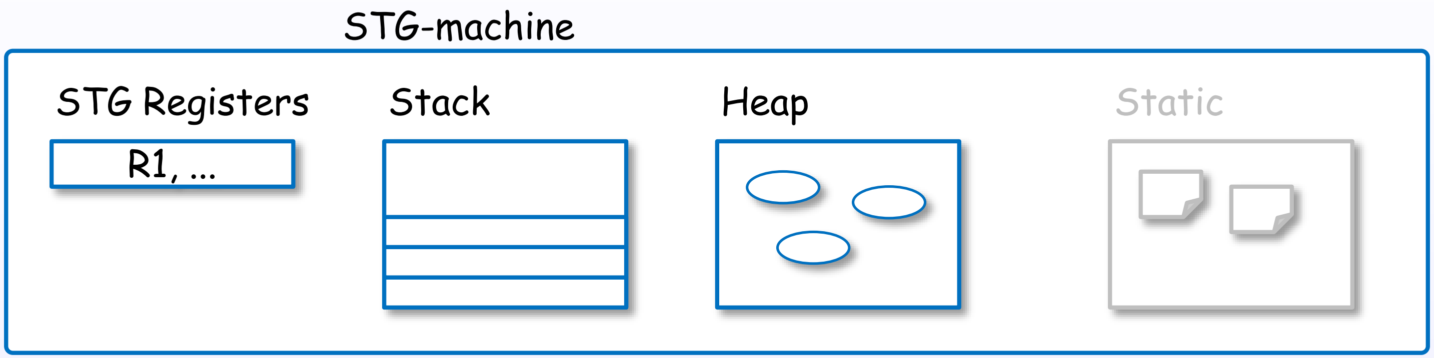


Evaluator (reducer, executer)
(abstract machine)



References : [H5], [H6], [H7], [D15]

STG-machine

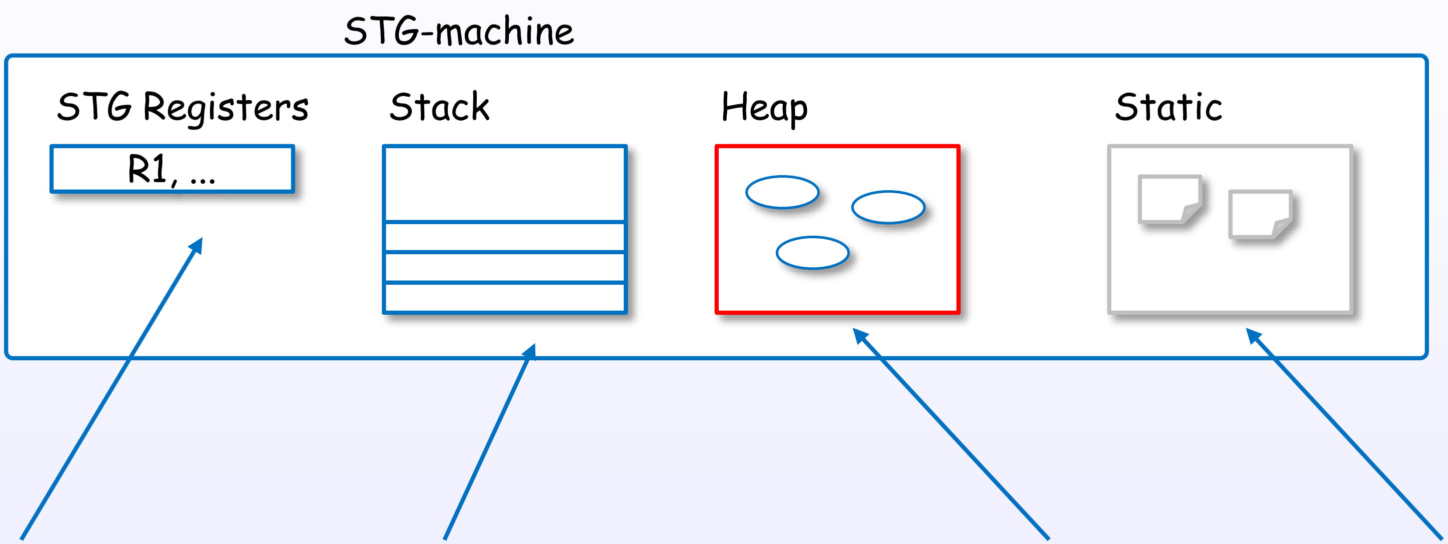


STG-machine is abstraction machine
which is defined by operational semantics.

STG-machine efficiently performs lazy graph reduction.

References : [H5], [H6], [H7], [D15]

STG-machine



mainly used for
call/return convention
various control

mainly used for
nest continuation
argument passing

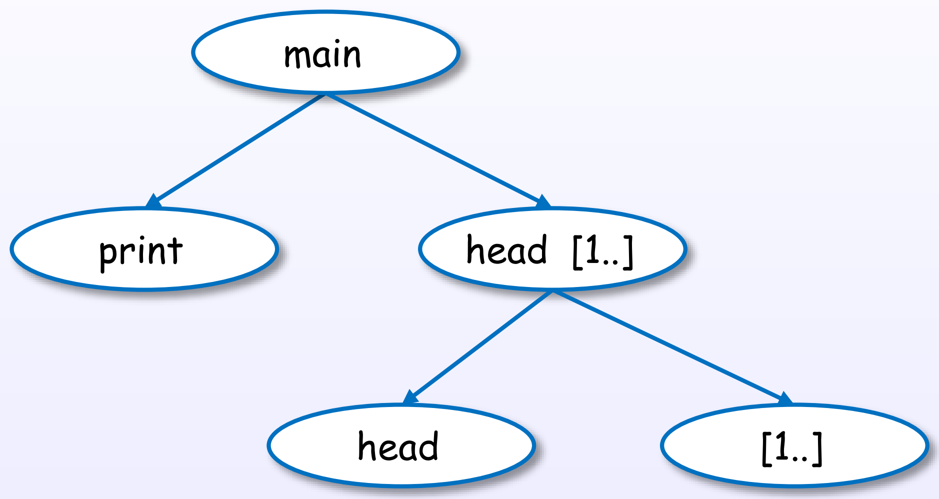
mainly used for
allocating objects
(thunks, datas,
functions)

mainly used for
code
static objects

References : [H5], [H6], [H7], [D15]

Example of mapping a code to a graph

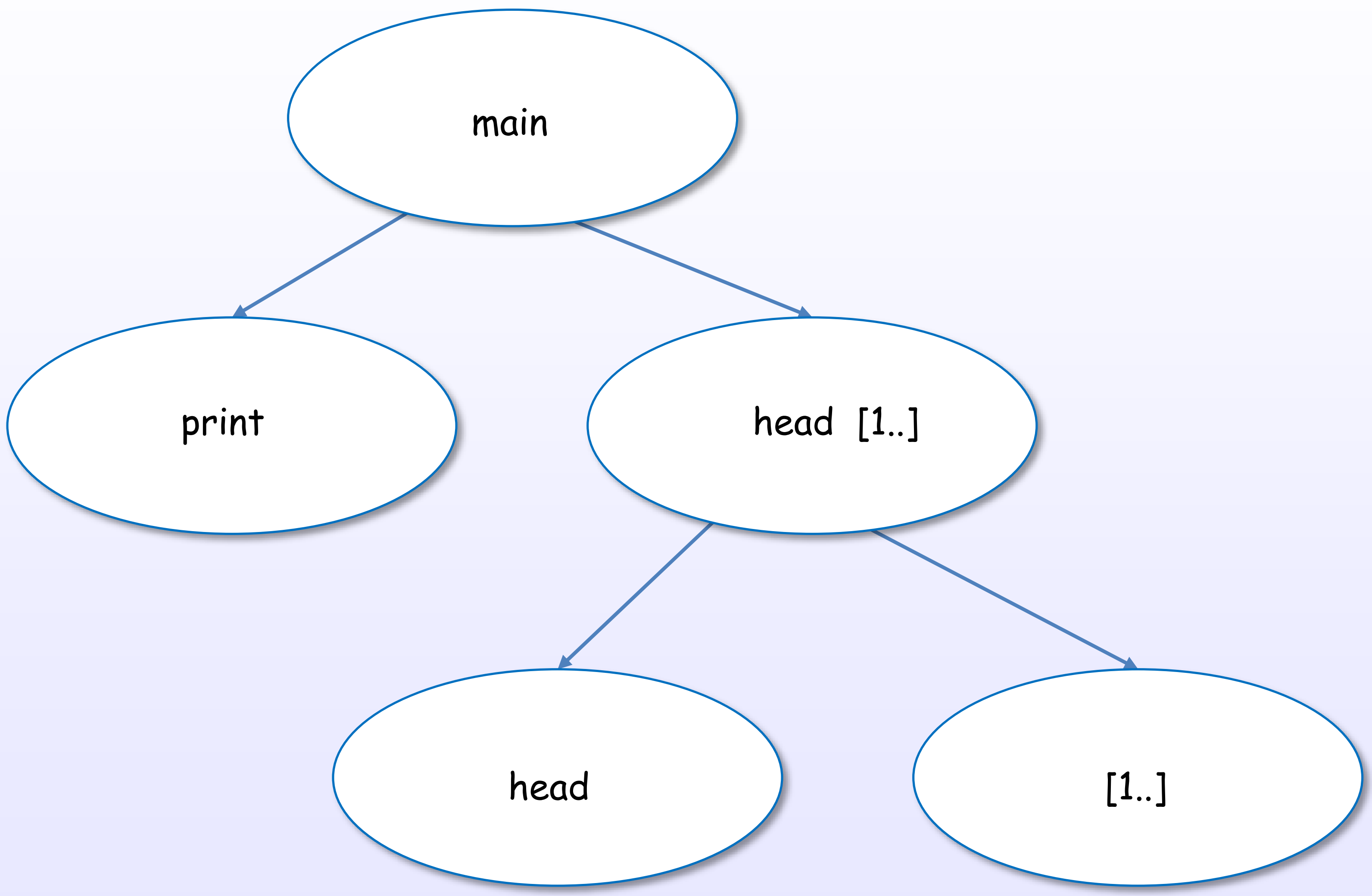
```
main = print (head [1..])
```



References : [H5], [H10]

Example of mapping a code to a graph

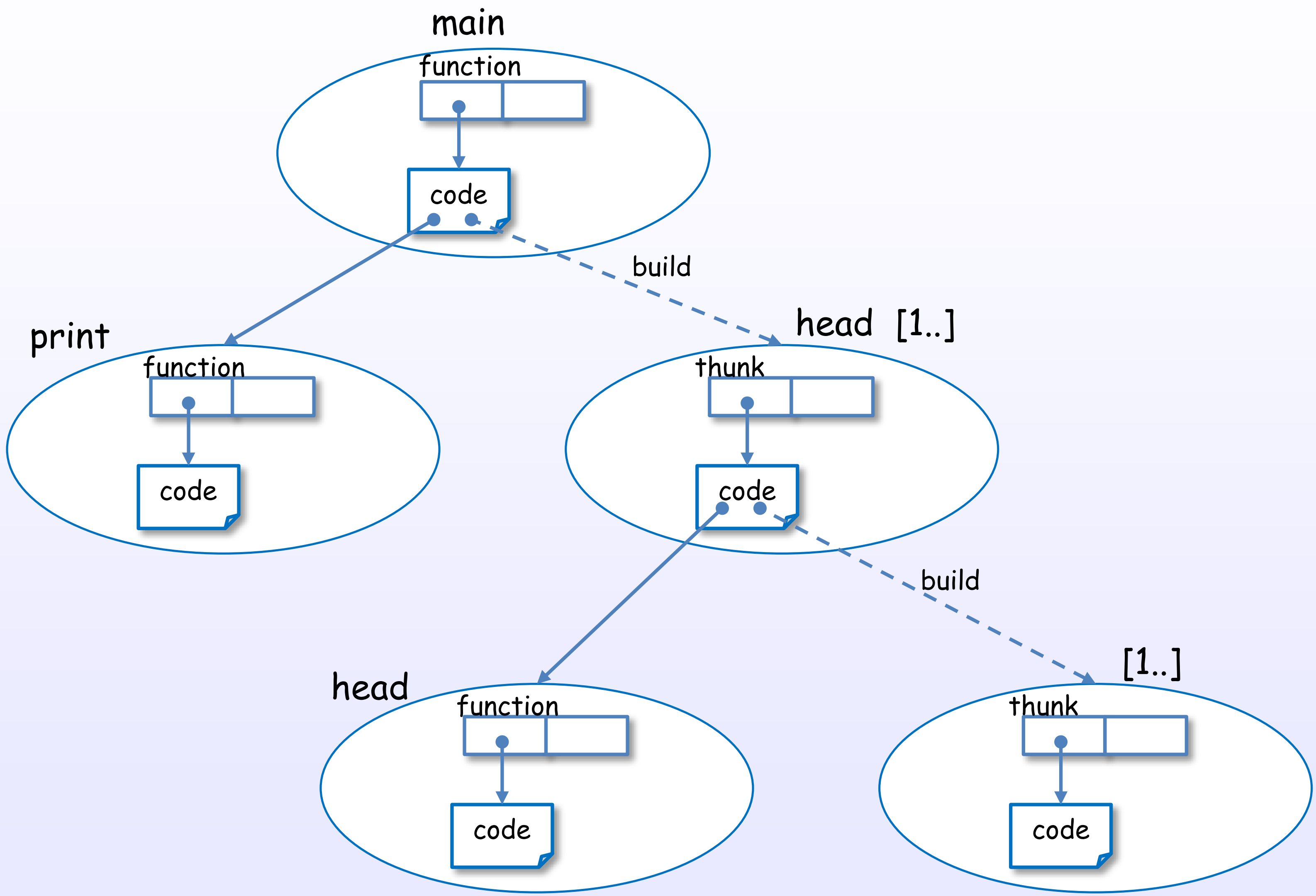
main = print (head [1..])



References : [H5], [H10]

Example of mapping a code to a graph

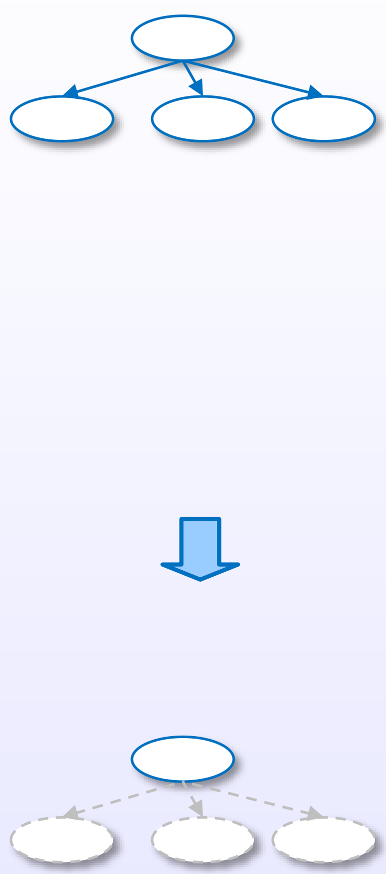
main = print (head [1..])



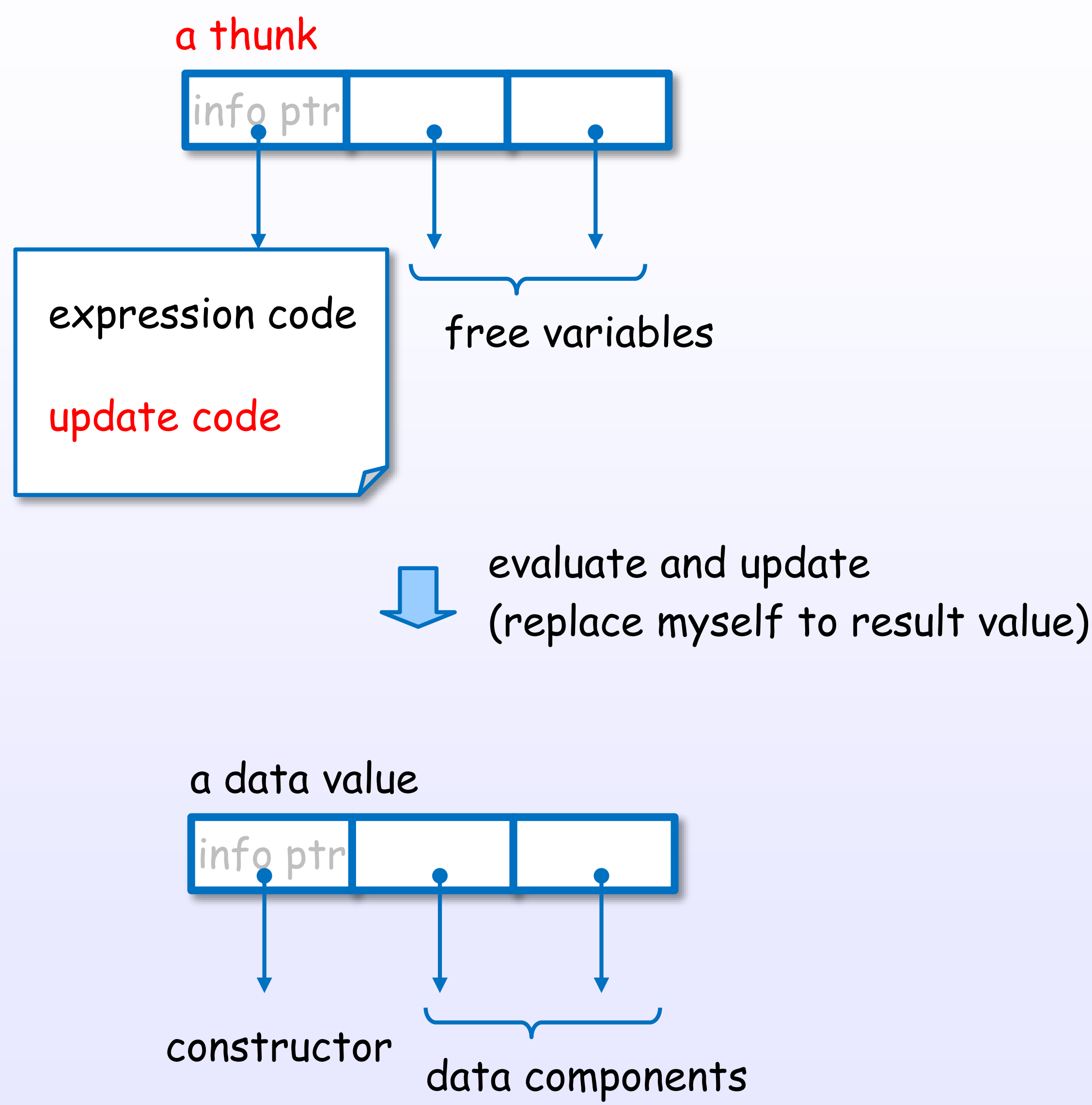
References : [H5], [H10]

Self-updating model

Expression

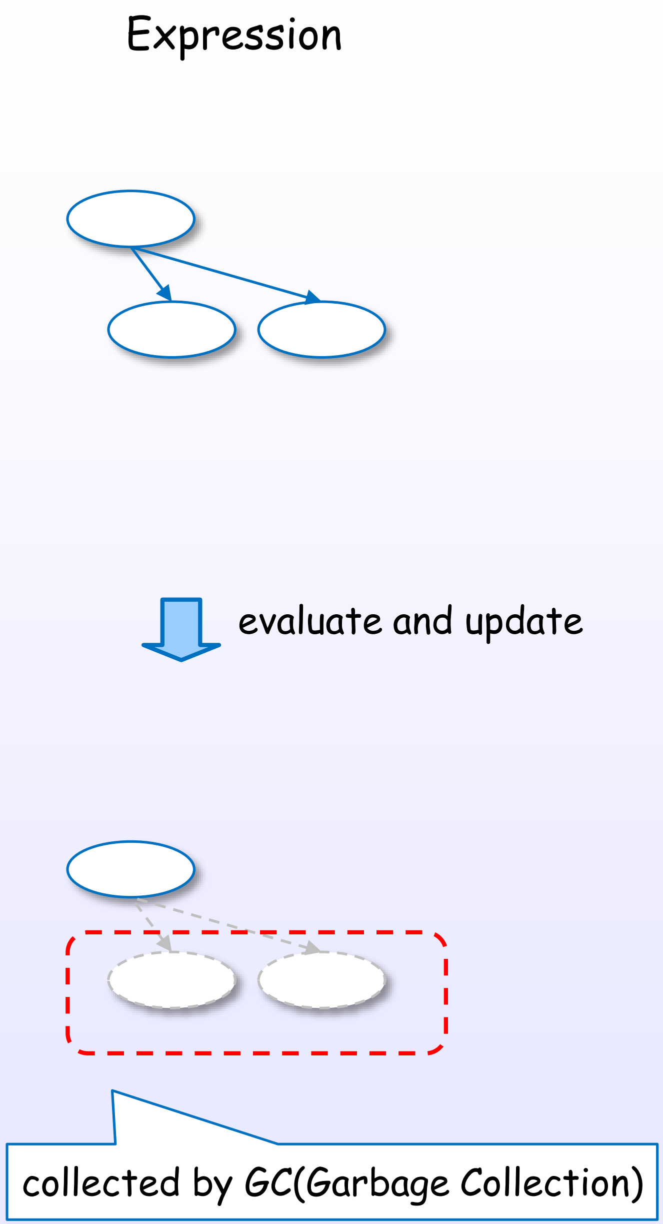


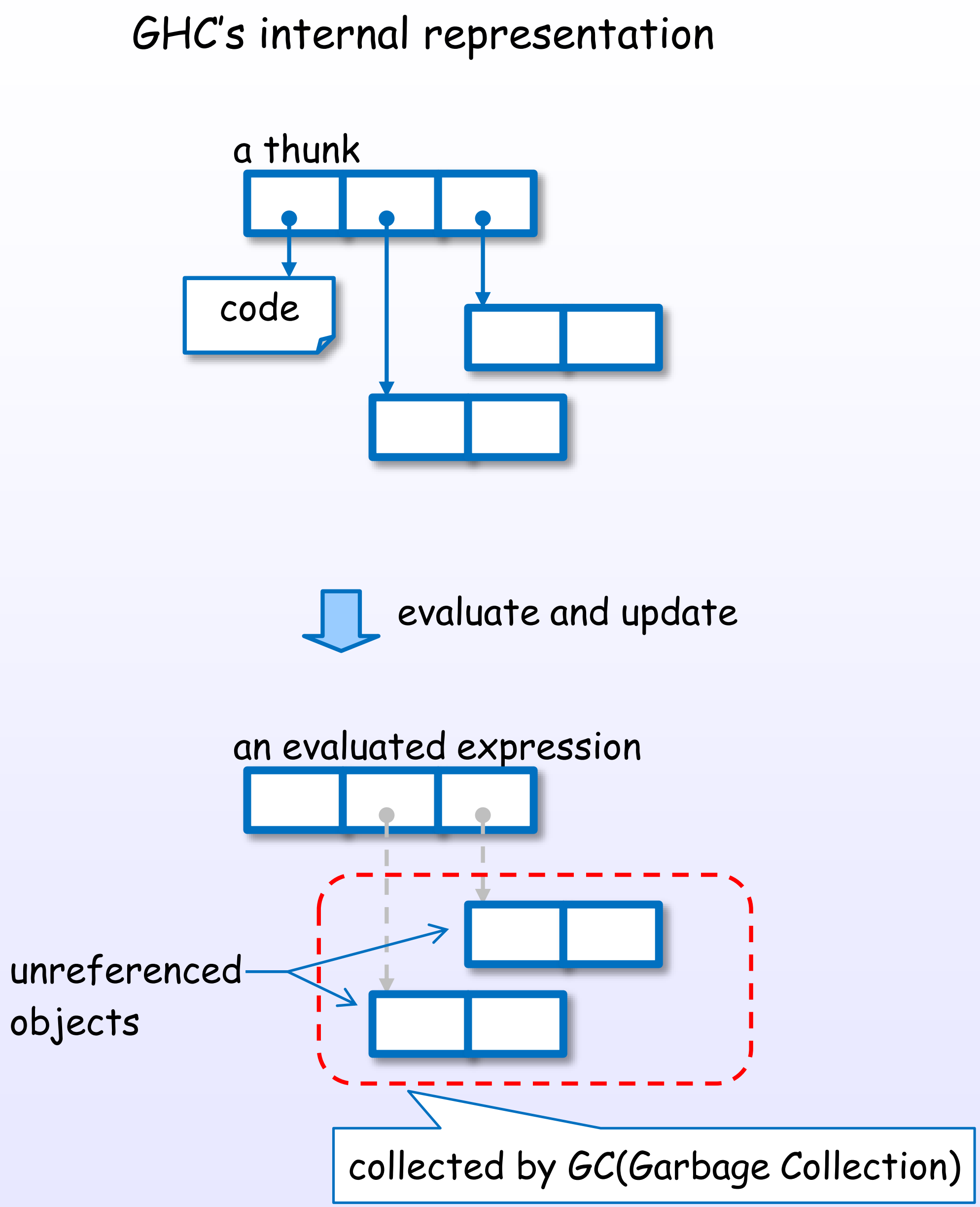
GHC's internal representation



References : [H5], [H6], [H7], [D15]

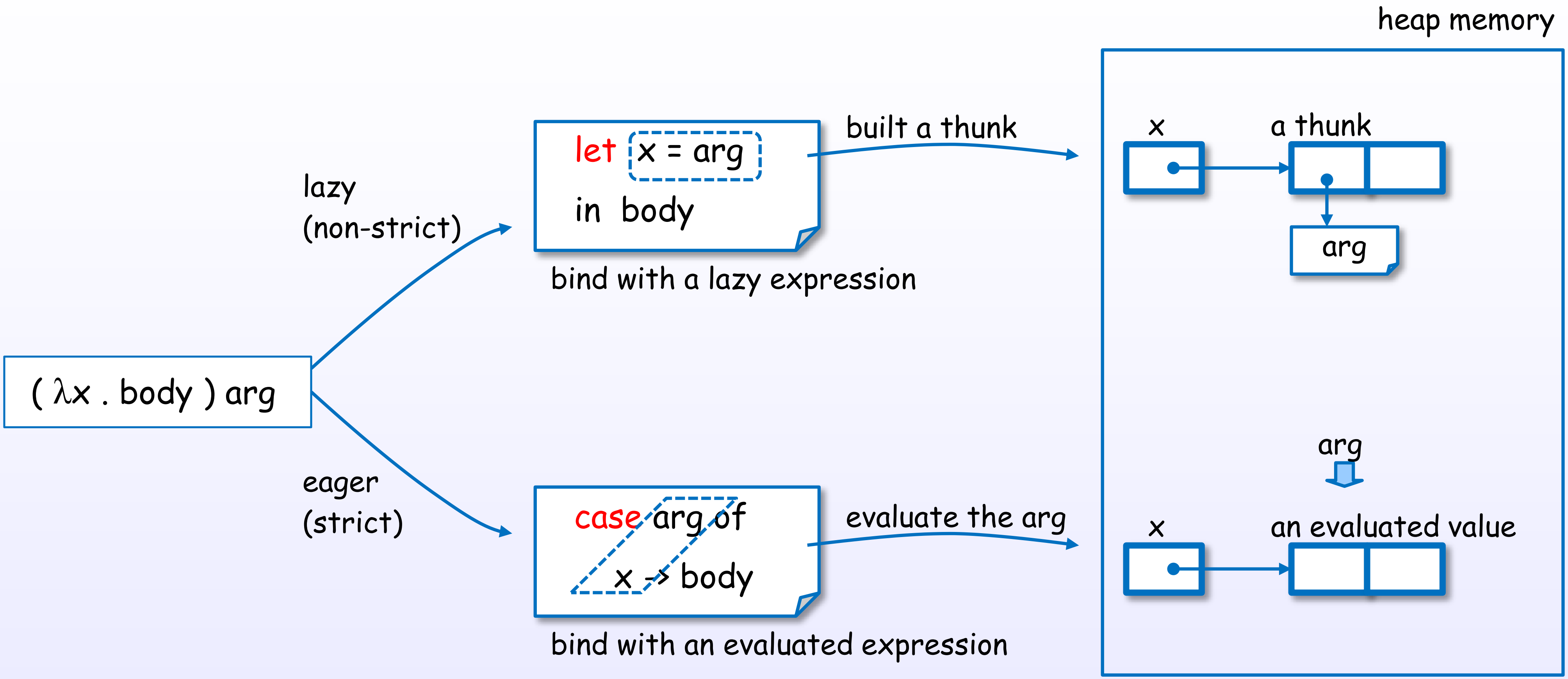
Unreferenced expressions (objects) will be removed by GC





References : [H5], [H6], [H4] Ch.12, [D15]

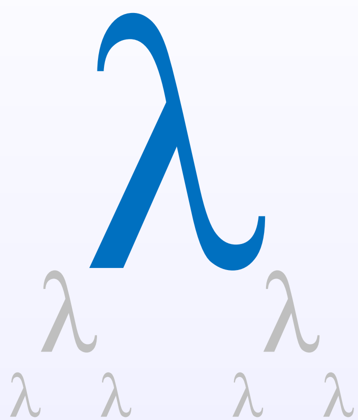
STG-machine associates directly ...



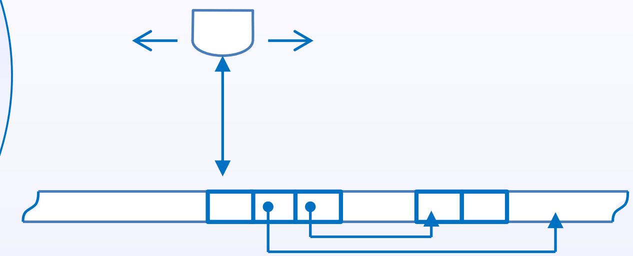
STG-machine associates directly lambda calculus and physical machine.

References : [H5], [H6], [H4] Ch.3

The STG-machine is ...



Lambda calculus



Turing machine

The STG-machine is the marriage of Lambda calculus and Turing machine.

STG-dump shows which expression is built as thunks

[Example.hs]

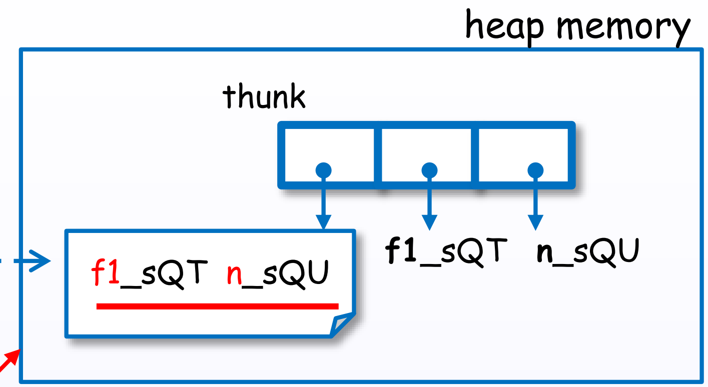
```
module Example where

fun  f1  n = take  1  (f1  n)
```

STG code dump
by "$ ghc -O -ddump-stg Example.hs"

```
Example.fun
  :: forall a_aME t_aMF. (t_aMF -> [a_aME]) -> t_aMF ->
[a_aME]
[GblId,
 Arity=2,
 Caf=NoCafRefs,
 Str=DmdType <L,1*C1(U)><L,U>,
 Unf=OtherCon []] =
    \r srt:SRT:[] [f1_sQT n_sQU]
      let {
        sat_sQV [Occ=Once, Dmd=<L,1*U>] :: [a_aMH]
        [LclId, Str=DmdType] =
            \s srt:SRT:[] [] f1_sQT n_sQU;
      } in  GHC.List.take_unsafe_UInt 1 sat_sQV;
```

let expression in STG language

build/allocate

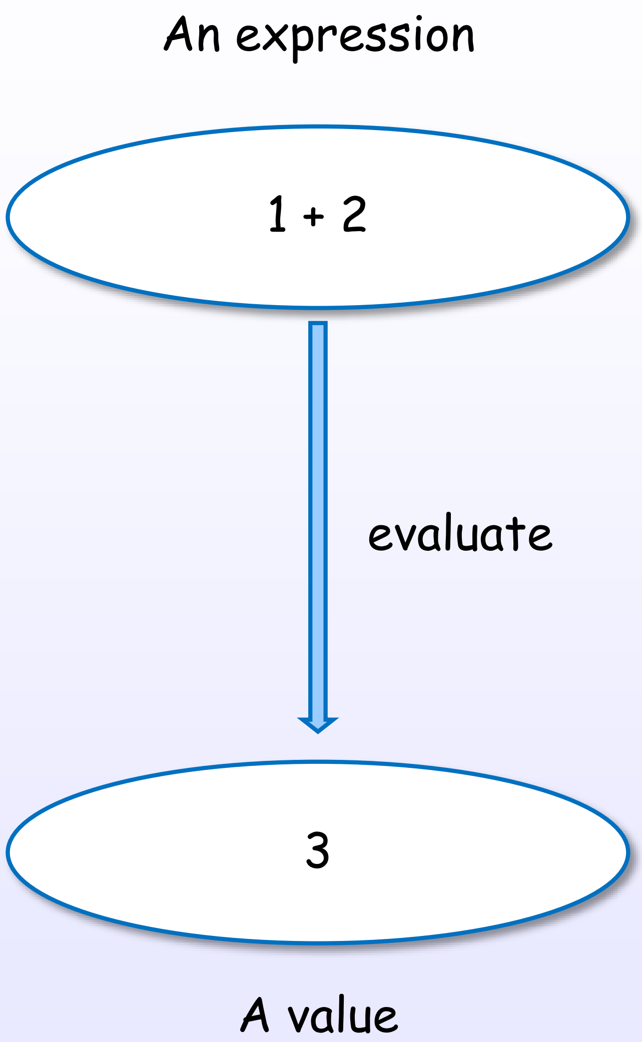


References : [H5], [H6], [H7], [D15]

6. Semantics

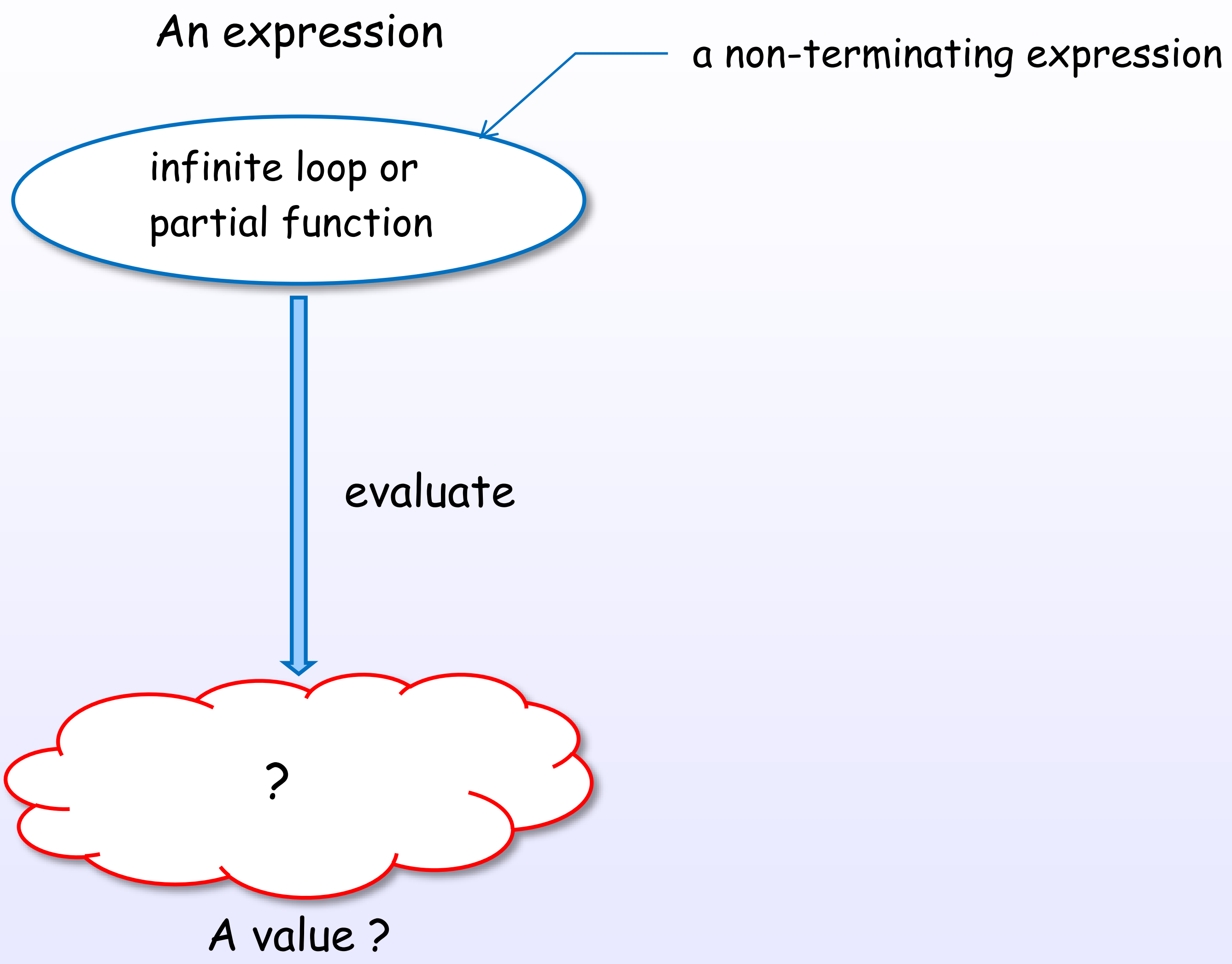
Bottom

A well formed expression should have a value



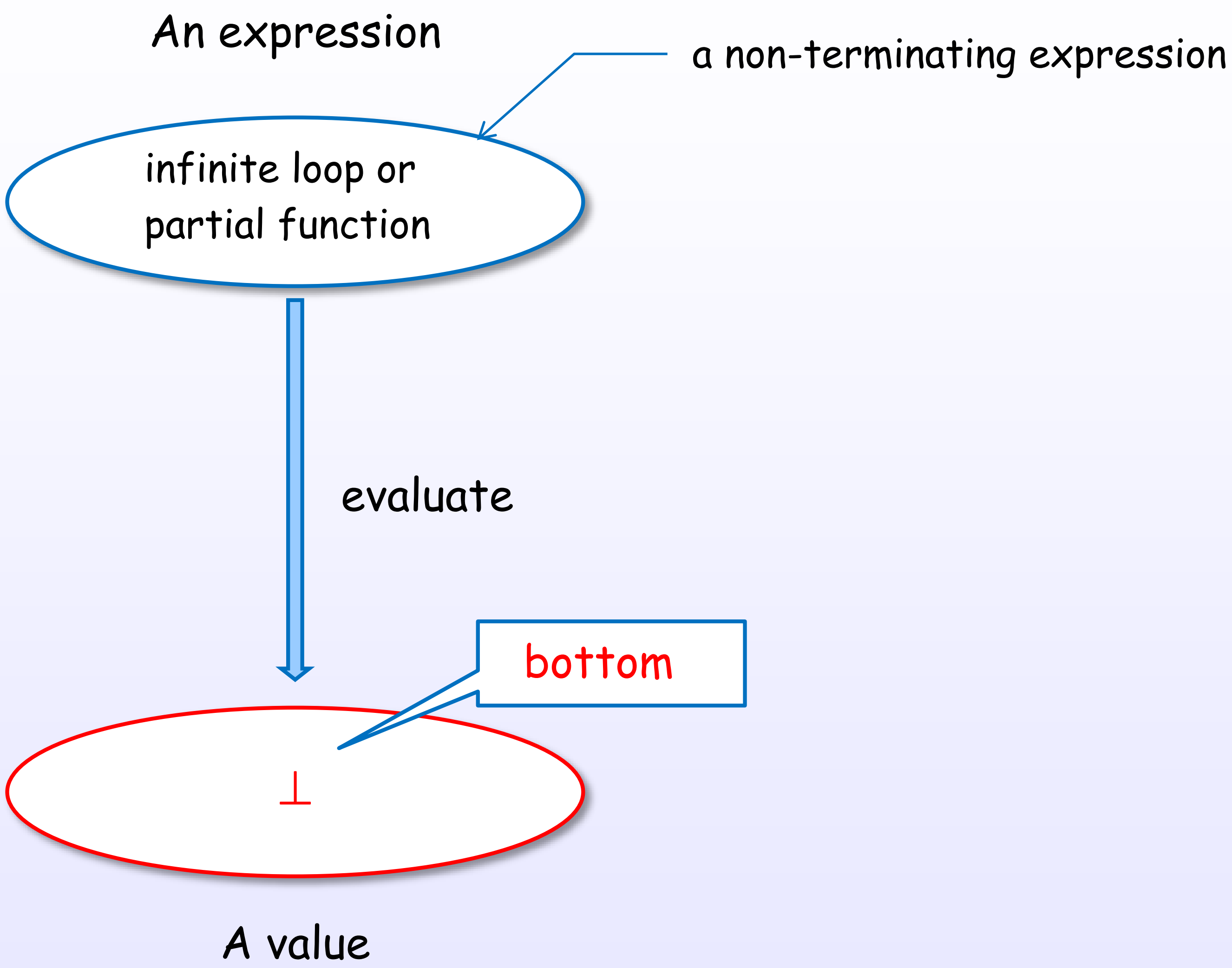
References : [B2] Ch.2, [W4]

What is a value in this case?



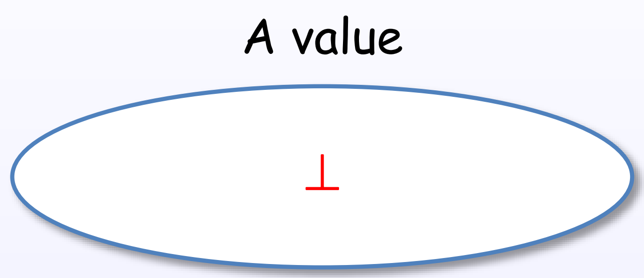
References : [B2] Ch.2, [H1] Ch.3, [W4], [H4] Ch.2, 22

A value “bottom” is introduced



References : [B2] Ch.2, [H1] Ch.3, [W4], [H4] Ch.2, 22

Bottom

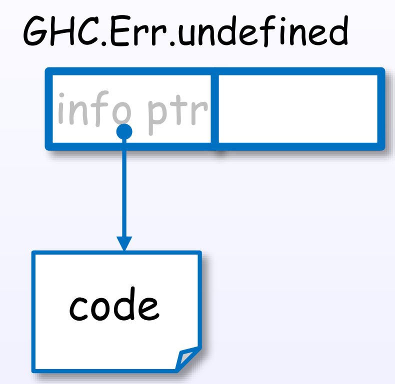


Bottom ($\perp$) is "an undefined value".

Bottom ($\perp$) is "a non-terminating value".

References : [B2] Ch.2, 9, [H1] Ch.3, [W4], [H4] Ch.2, 22

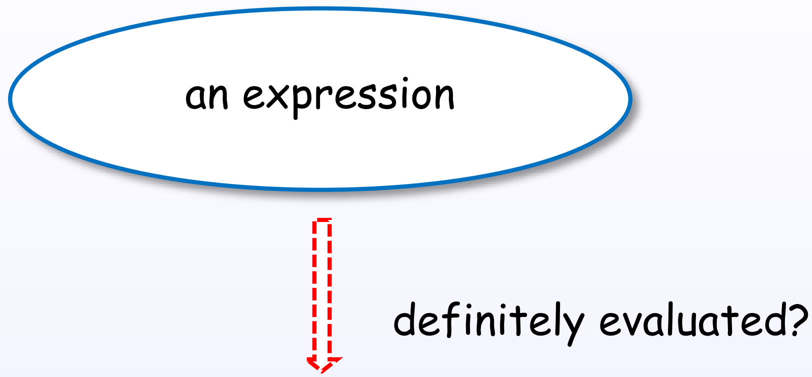
“undefined” function represents “bottom” in GHC

Haskell code	Expression	GHC’s internal representation
undefined :: a	$\perp$	

References : [B2] Ch.2, 9, [H1] Ch.3, [W4], [H4] Ch.2, 22

Strict/Non-strict

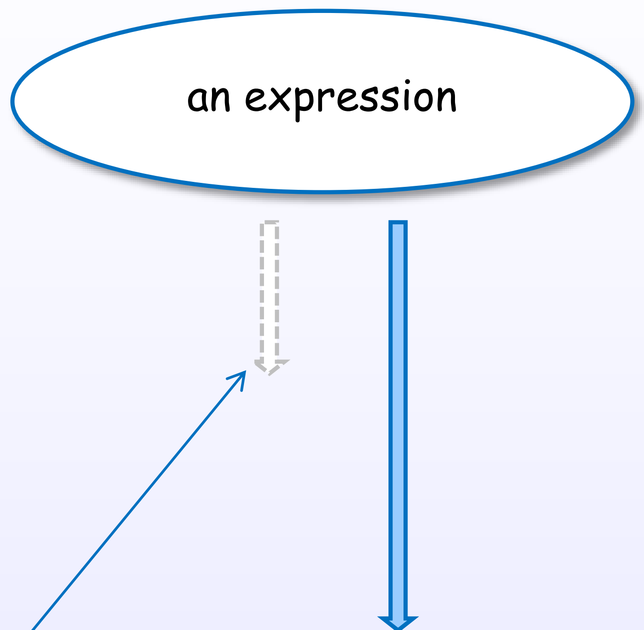
Strictness



Strictness is “evaluation demand” of the expression.

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22, [H15], [H16]

Strict and non-strict

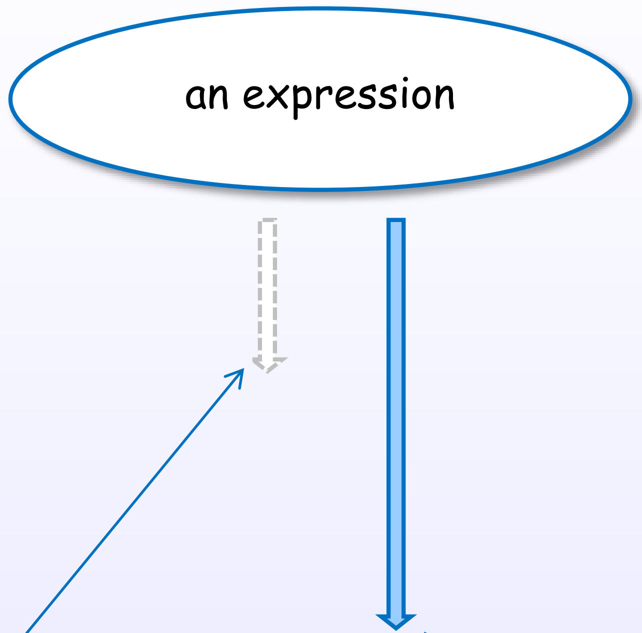


"Non-strict" means that
the expression may or may not be evaluated.

"Strict" means that
the expression is definitely evaluated.

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22, [H15], [H16]

Strict and non-strict



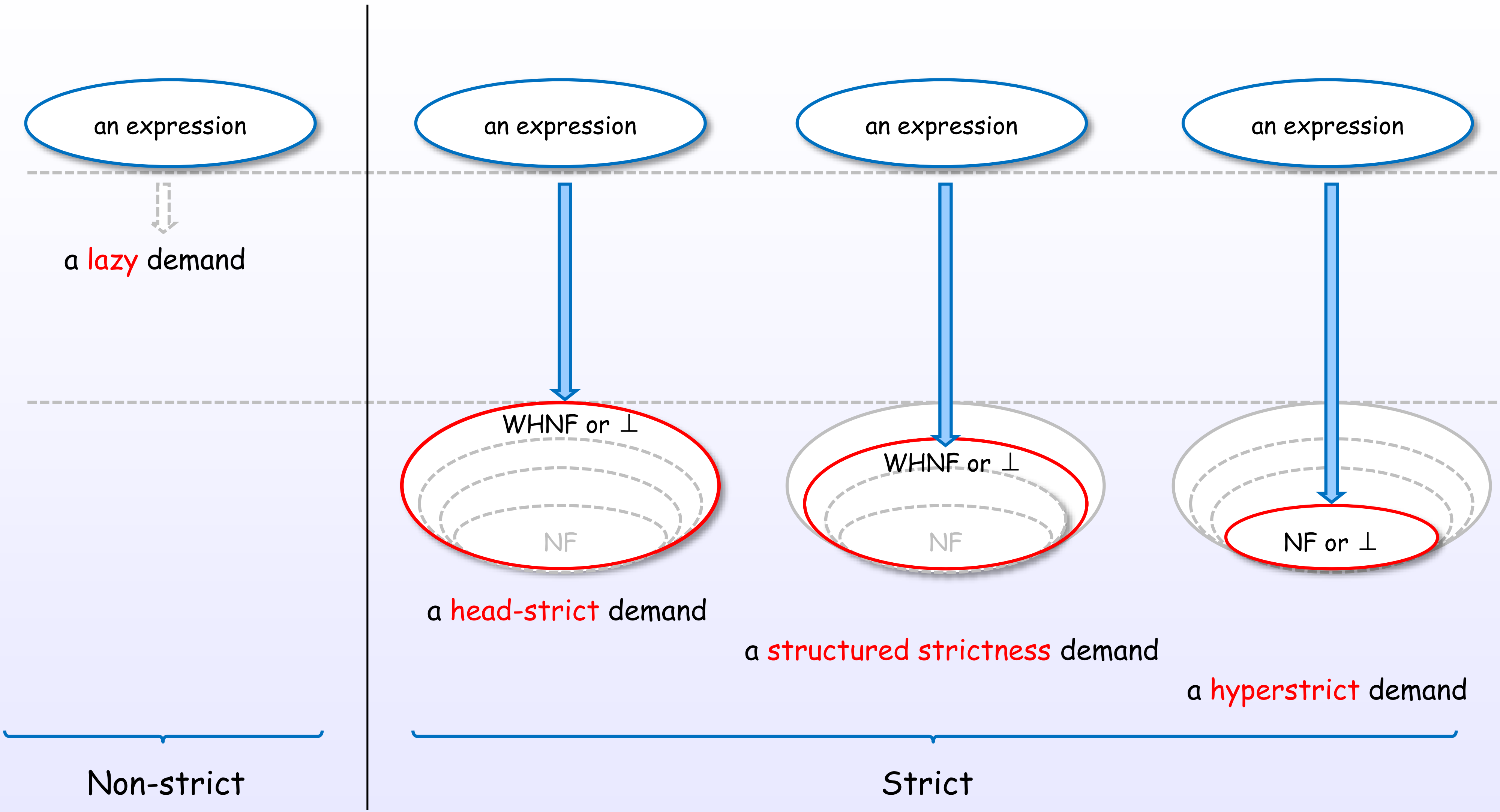
"Non-strict" means that
the expression may or may not be evaluated.

"Strict" means that
the expression is definitely evaluated.

GHC implements non-strict semantics by lazy evaluation.

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22, [H15], [H16]

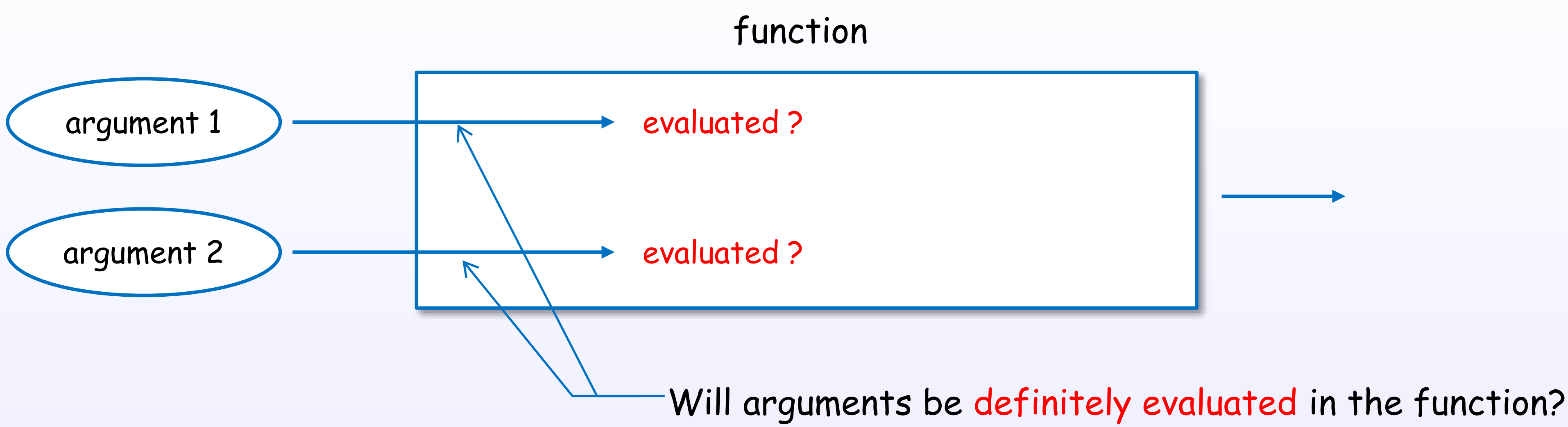
GHC has the lattice of strictness



There are multiple levels in strict.

References : [H15], [H16], [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

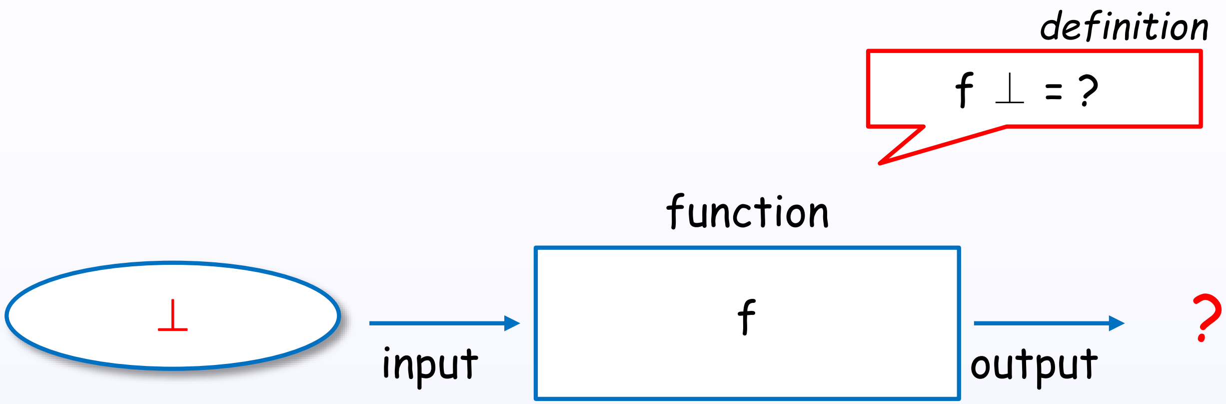
Strictness of a function



A function places "strictness demands" on each of its arguments.

References : [H15], [H16], [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Strictness of a function is formally defined

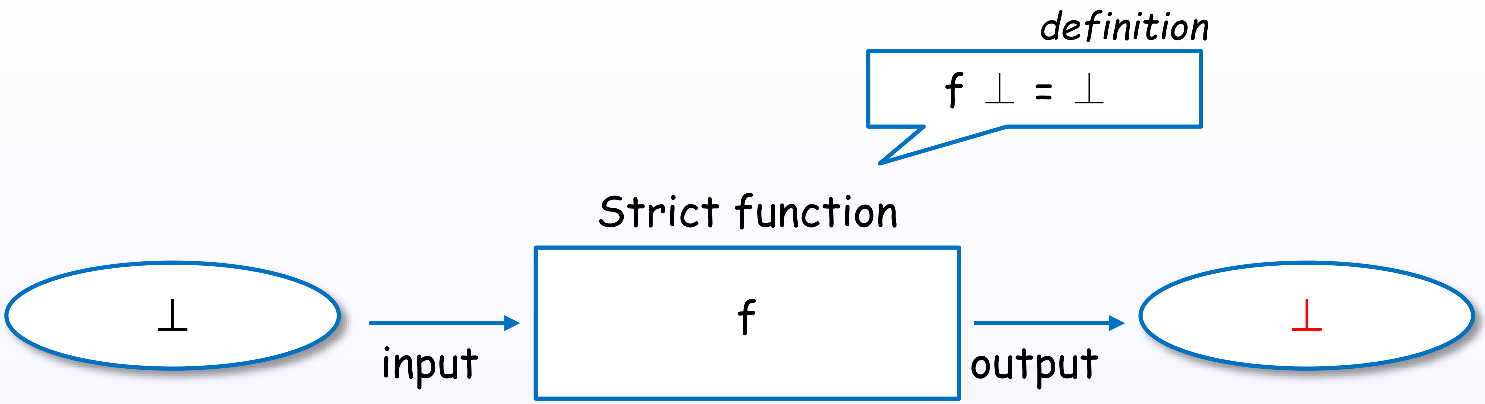


Strictness of a function can be defined with the association between input and output.

"given a non-terminating arguments, the function will terminate?"

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Definition of the strict function

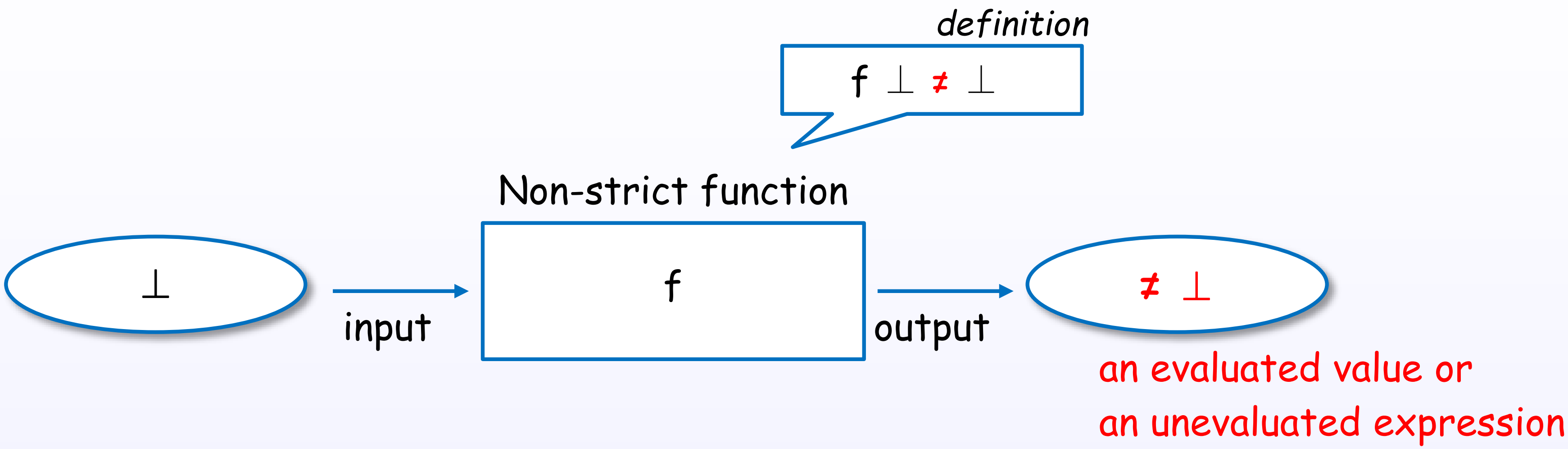


Strict function's output is bottom when input is bottom.

given a non-terminating arguments, strict function will not terminate.

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Definition of the non-strict function



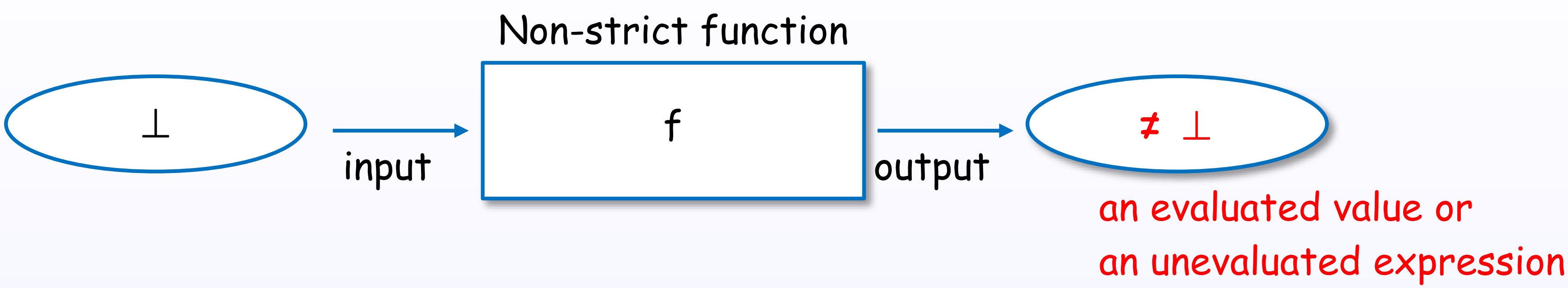
Non-strict function's output is not bottom when input is bottom.

given a non-terminating arguments, non-strict function will terminate.

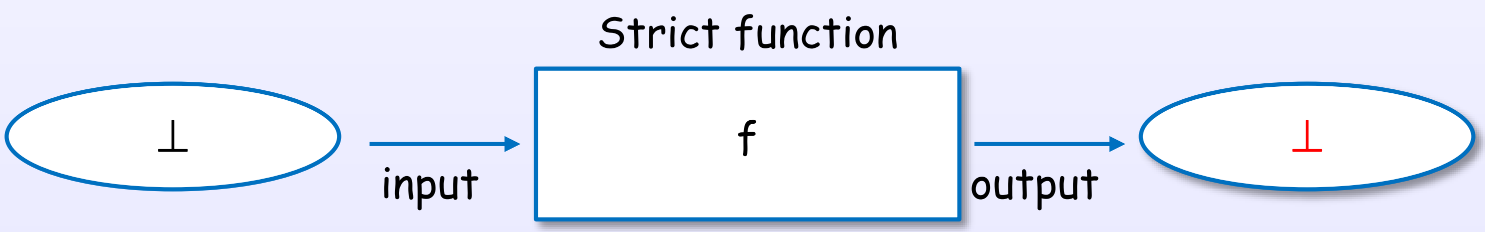
References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Strict and Non-strict functions

Non-strict

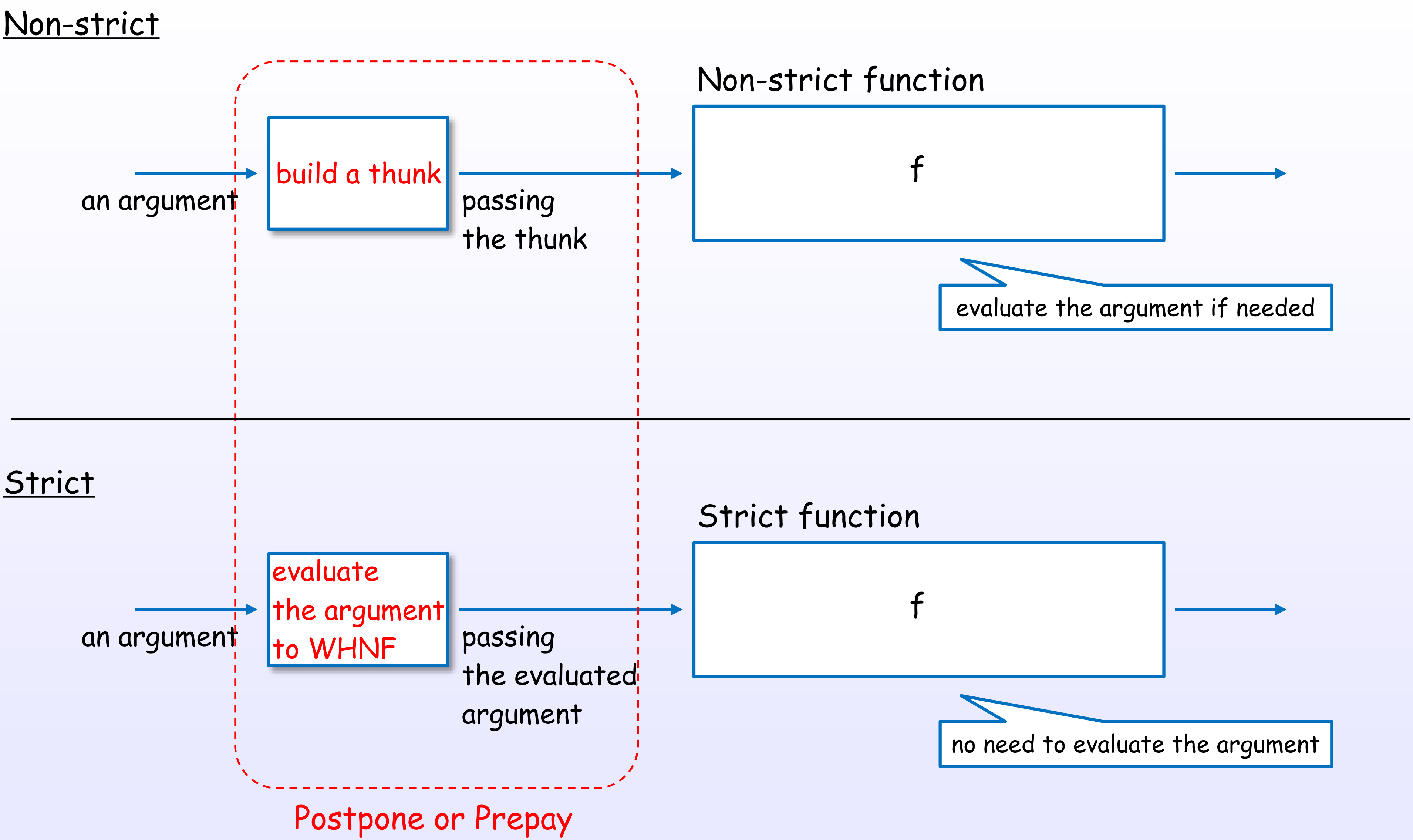


Strict



References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Function application and strictness



The front stage is also important.

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Strict and normal form

Example of function application

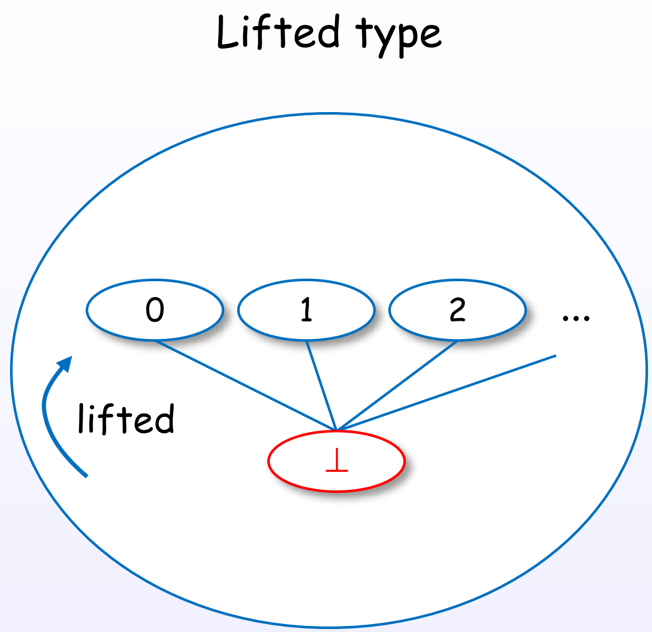
	to WHNF	to NF
Non-strict	f $ arg	
Strict	f $! arg (seq)	f $!! arg (deepseq)

Strict ≠ Normal form

References : [B2] Ch.2, [W1], [W4], [H4] Ch.2, 22

Lifted and boxed types

Lifted types

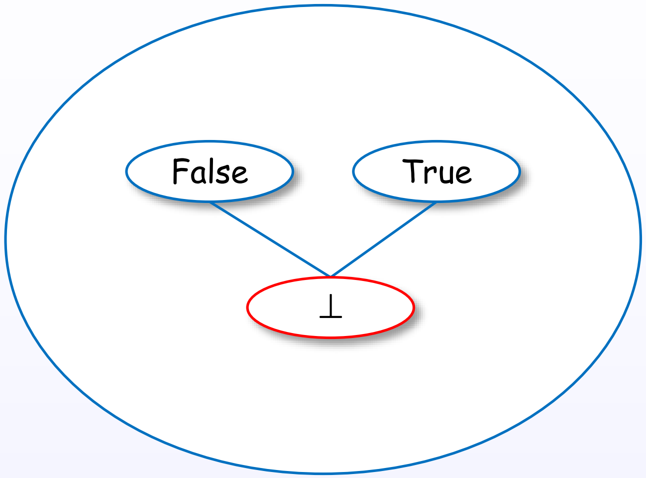


Lifted types include bottom as an element.

References : [W4], [B2], [H14]

Lifted type's declaration implicitly include bottom

Bool type

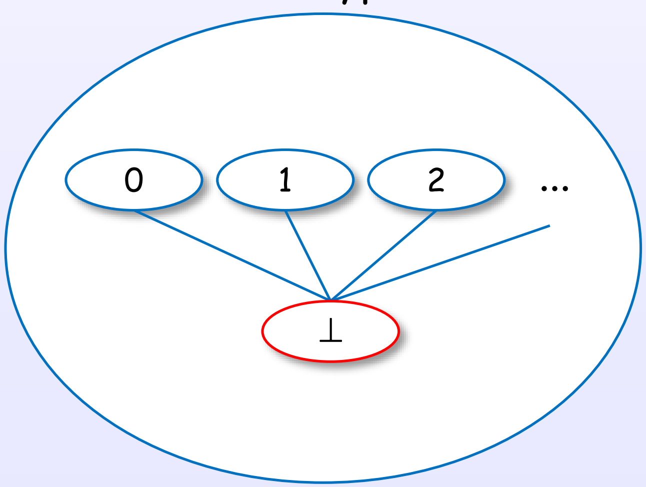


data declaration

```
data  Bool    =   False
              |   True
              |   ⊥
```

implicitly included

Int type

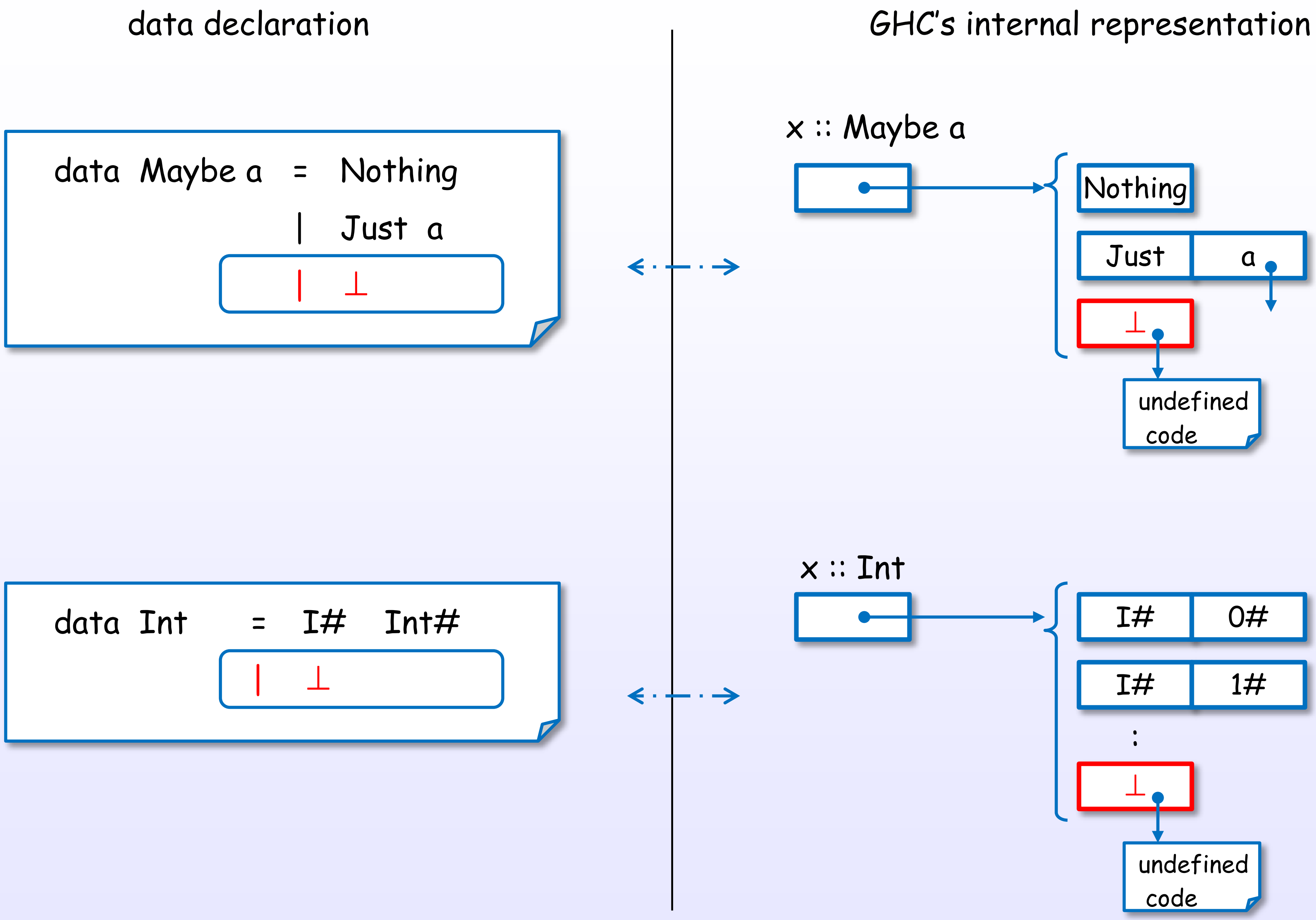



```
data  Int     =   I#   Int#
              |   ⊥
```

implicitly included

References : [W4], [B2], [H14]

Lifted type are also implemented by uniform representation

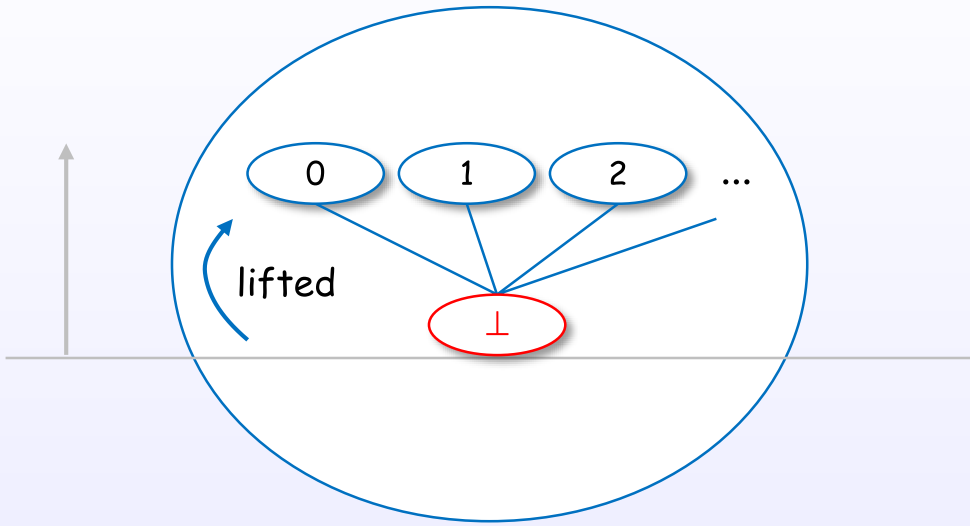


References : [W4], [B2], [H14]

Lifted and unlifted types

Lifted types

Int type

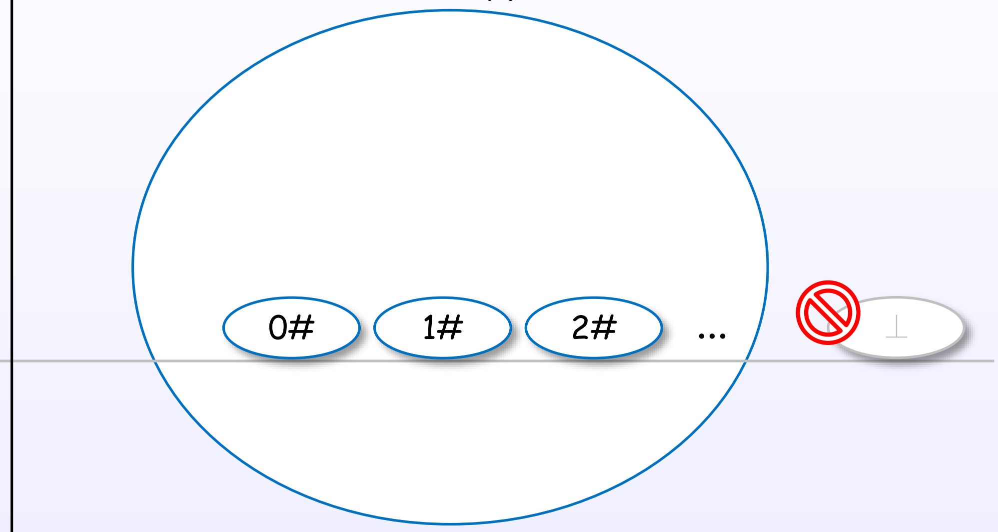


Lifted types include bottom.

(Bool, Int, Char, Maybe, List, ...)

Unlifted types

Int# type



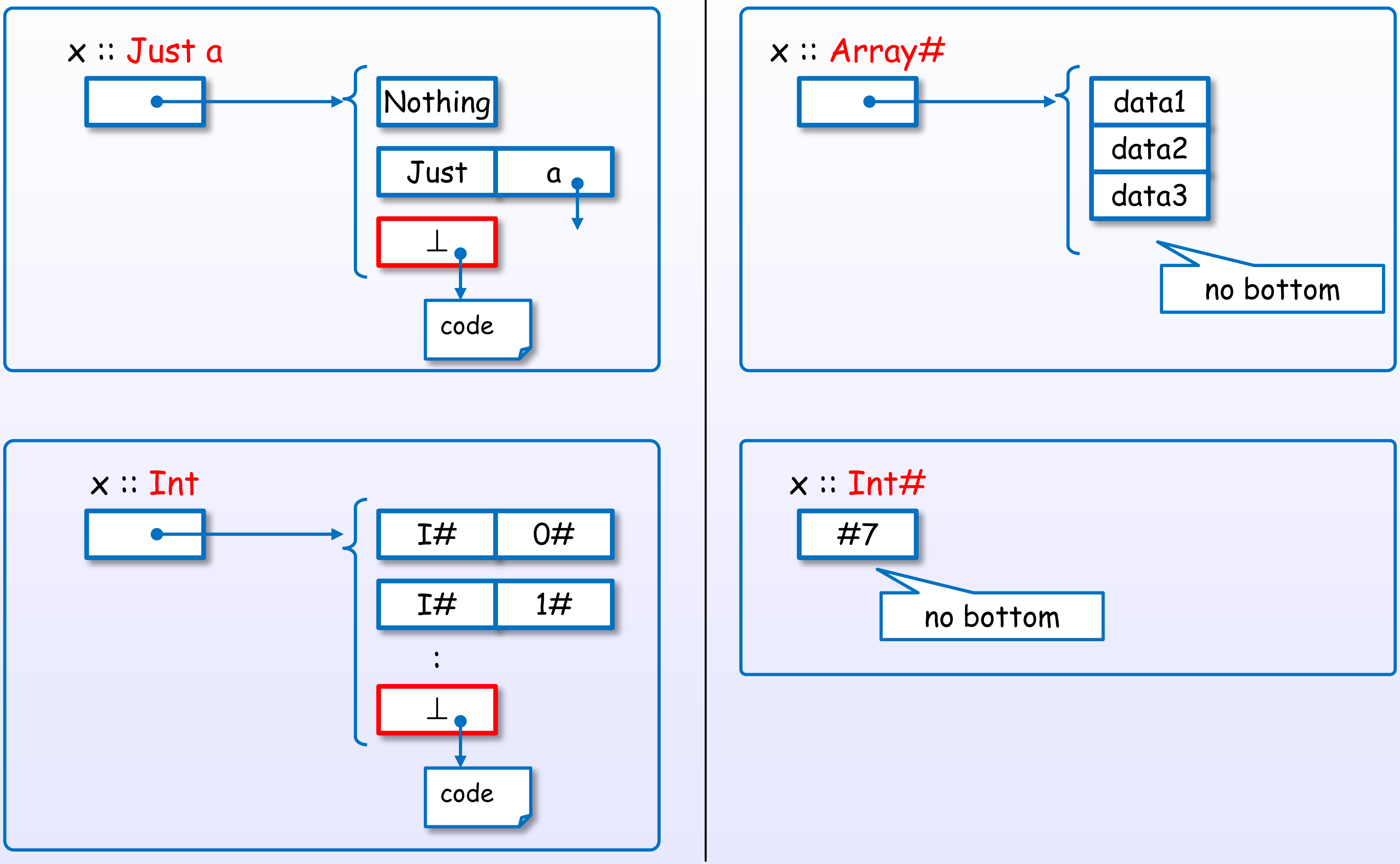
Unlifted types do not include bottom.

(Int#, Char#, Addr#, Array#, ByteArray#, ...)

References : [W4], [B2], [H14]

Example of lifted and unlifted types

Lifted types | Unlifted types



References : [W4], [B2], [H14]

Boxed and unboxed types

Boxed types

Boxed types are represented as a pointer.

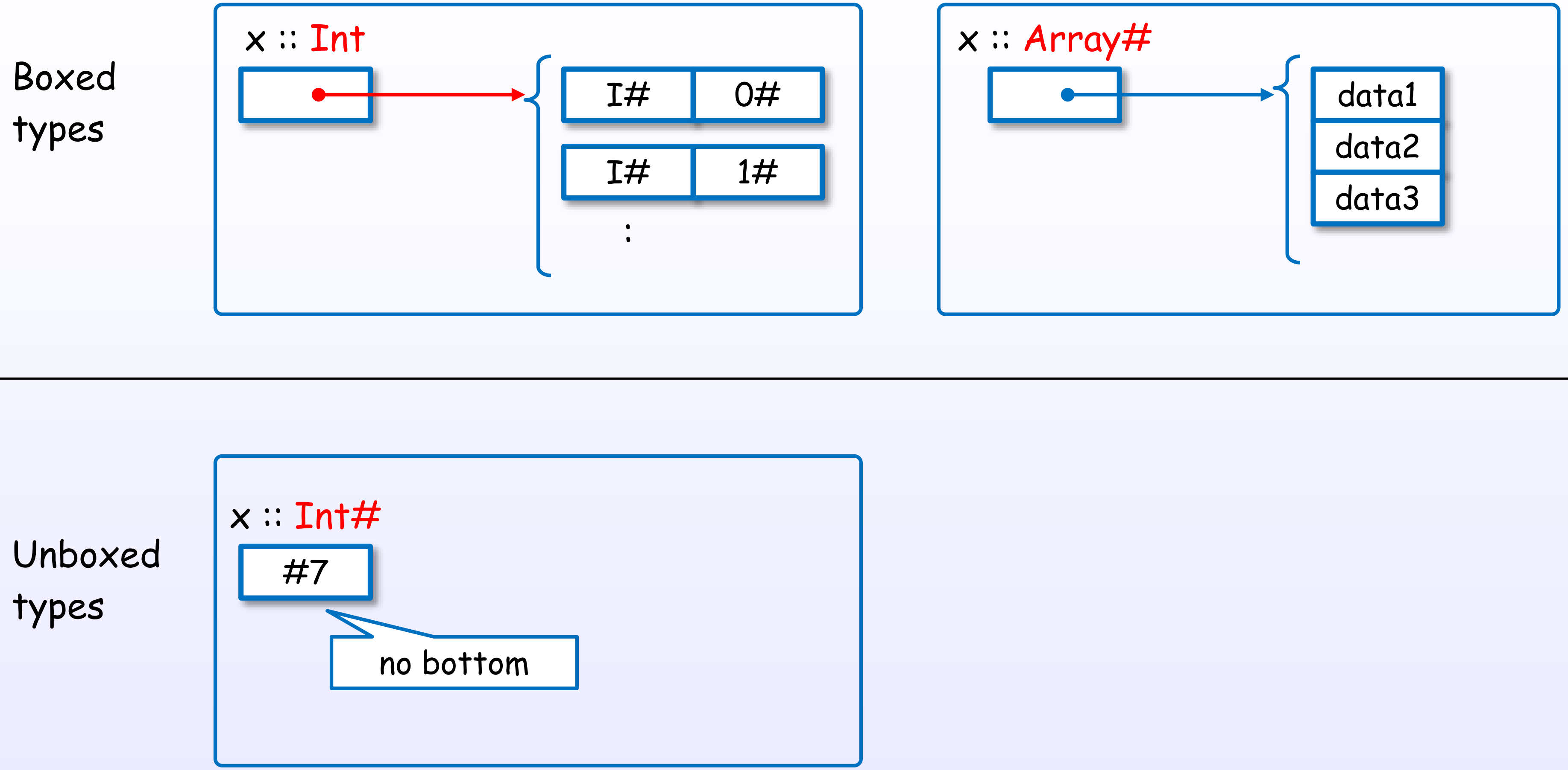
Unboxed types

Unboxed types are represented other than a pointer.

- no bottom (can't be lifted)
- no thunk (can't be postponed)
- no polymorphism (non-uniform size)
+ low cost memory size (no pointer)
+ high performance (no wrap/unwrap)

References : [W4], [B2], [H14]

Example of boxed and unboxed types



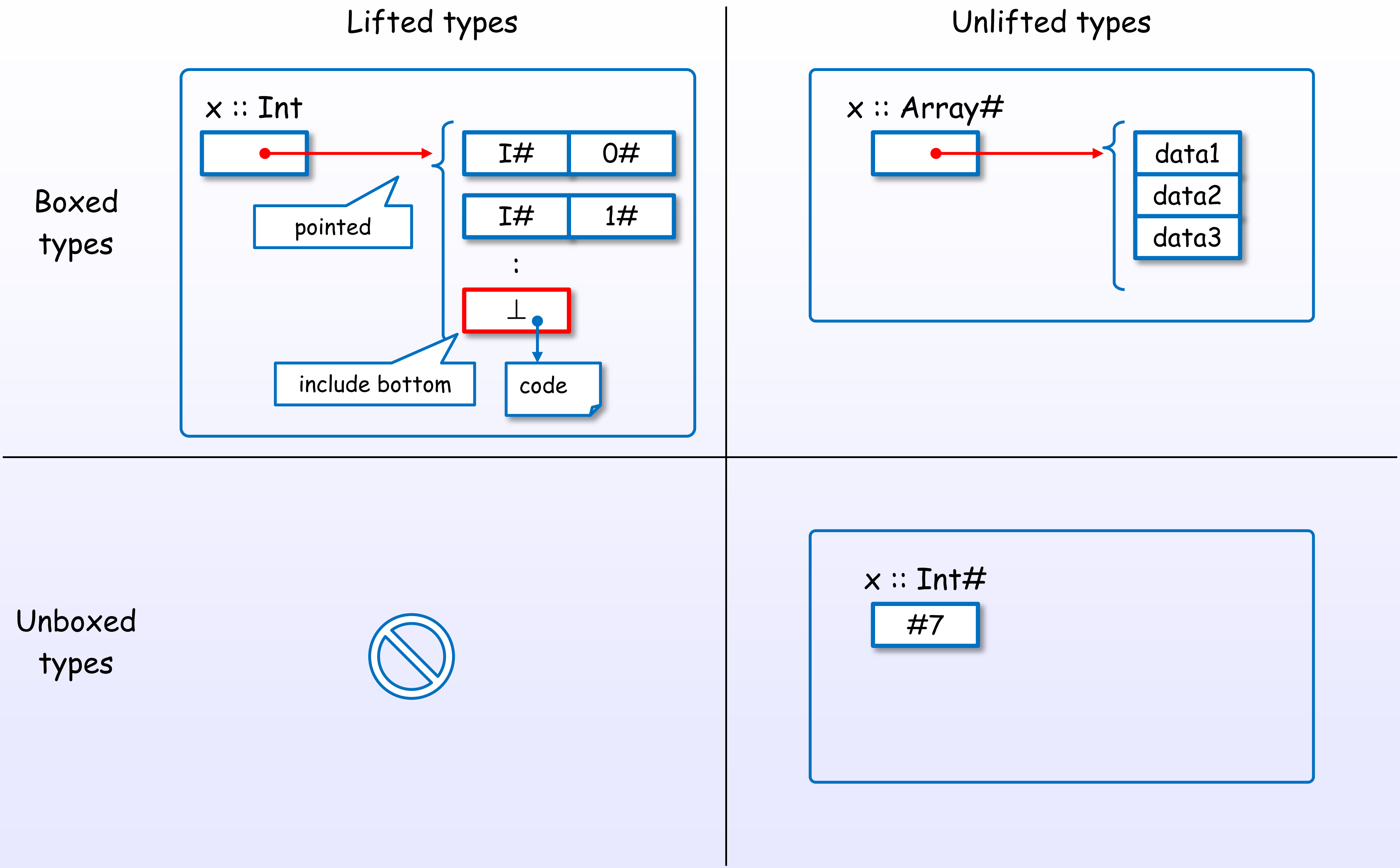
References : [W4], [B2], [H14]

Lifted and boxed types

	Lifted types	Unlifted types
Boxed types	Int Char Float Maybe :	Array# ByteArray# : no bottom
Unboxed types	unboxed can't be lifted	Int# Char# Float# : no bottom no packed

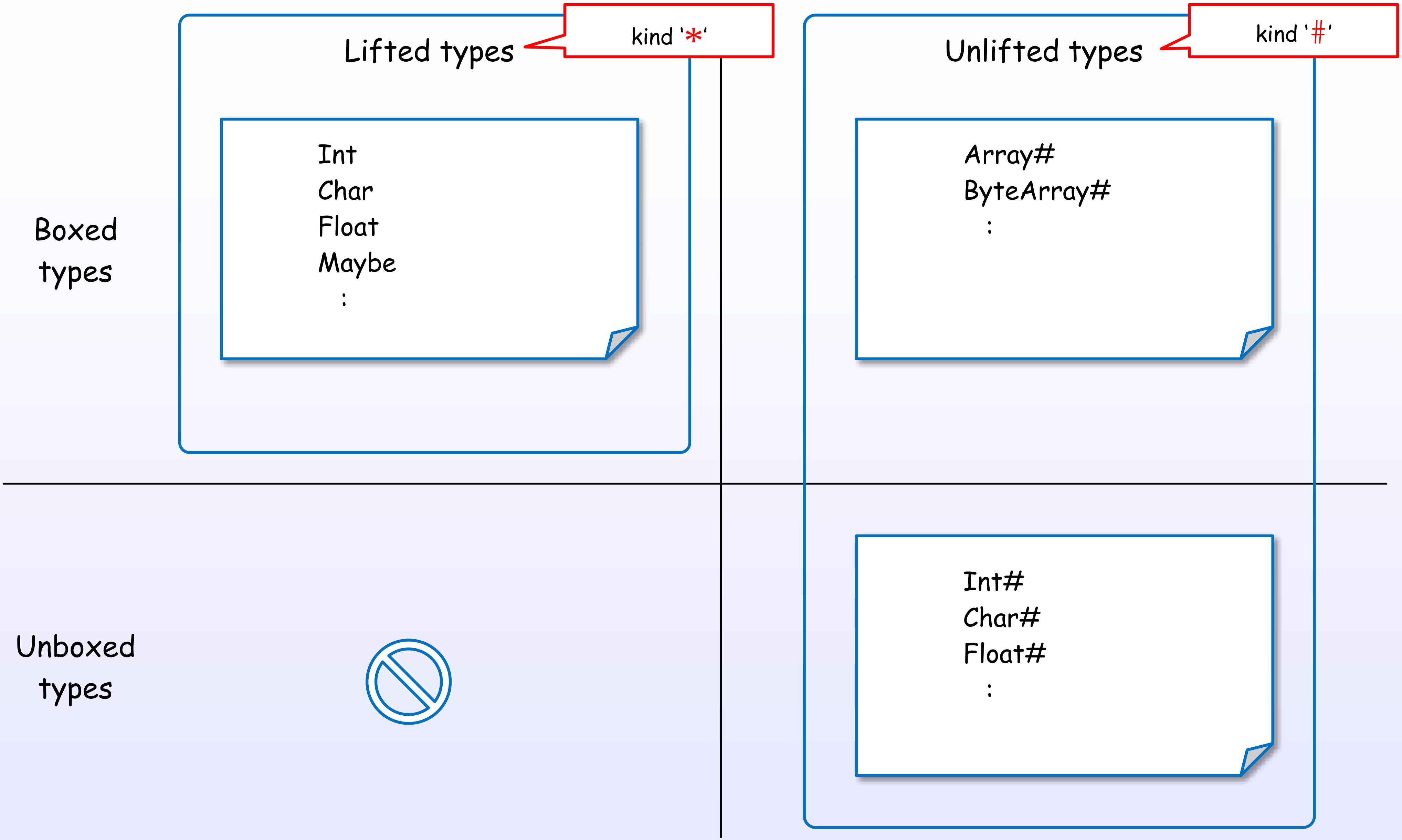
References : [W4], [B2], [H14]

Example of lifted and boxed types



References : [W4], [B2], [H14]

Types and kinds

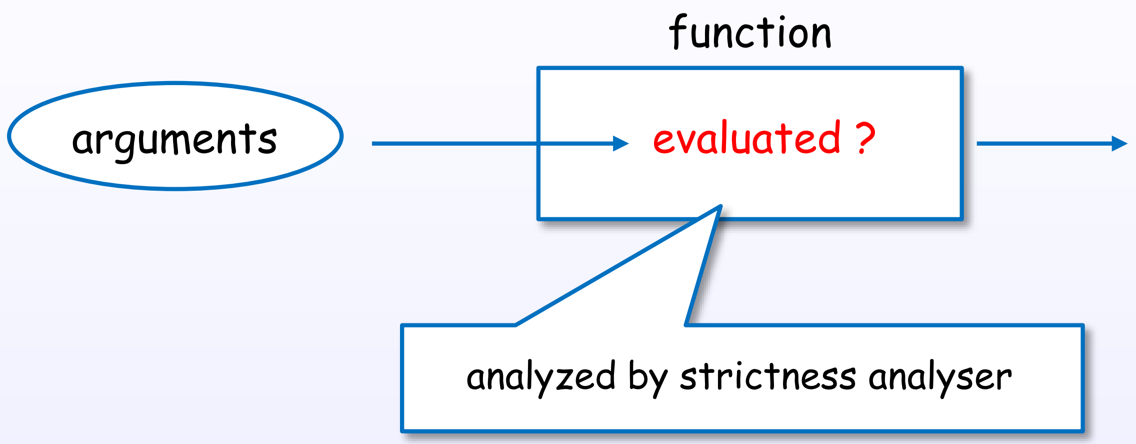


Note:
Identifier's '#' customarily means "primitive" rather than "unboxed" or "unlifted".
Kind's '#' means "unlifted".

References : [B6] Ch.29, [W4], [B2], [H14]

Strictness analysis

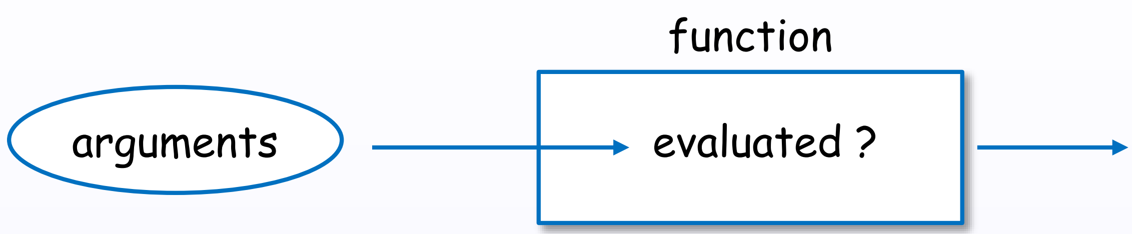
Strictness analysis



Strictness analysis analyzes whether a function is sure to evaluate its argument.

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H16], [H13], [H2]

Strictness analysis in GHC



demand analyser

Strictness demands

- a lazy demand
- a head-strict demand
- a structured strictness demand
- a hyperstrict demand

Absence/usage demands

- a used demand
- a head-used demand
- a structured used demand
- unused (absence)

Demand analysis (= Strictness analysis + Absence analysis + ...)

GHC's demand analyser implements strictness analysis.

References : [H15], [H16], [H2], [H4] Ch.22, [H8], [W6], [W3], [H13]

Example of strictness analysis information in GHC

[Example.hs]

```
module Example where

f1 :: Bool -> Int -> Maybe Int
f1  c  n = case c of
          True  -> Just n
          False -> Nothing
```

Strictness analysis dump
by "$ ghc -O -ddump-strsigs Example.hs"

```
==================== Strictness signatures ====================

Example.f1: <S,1*U><L,U>
```

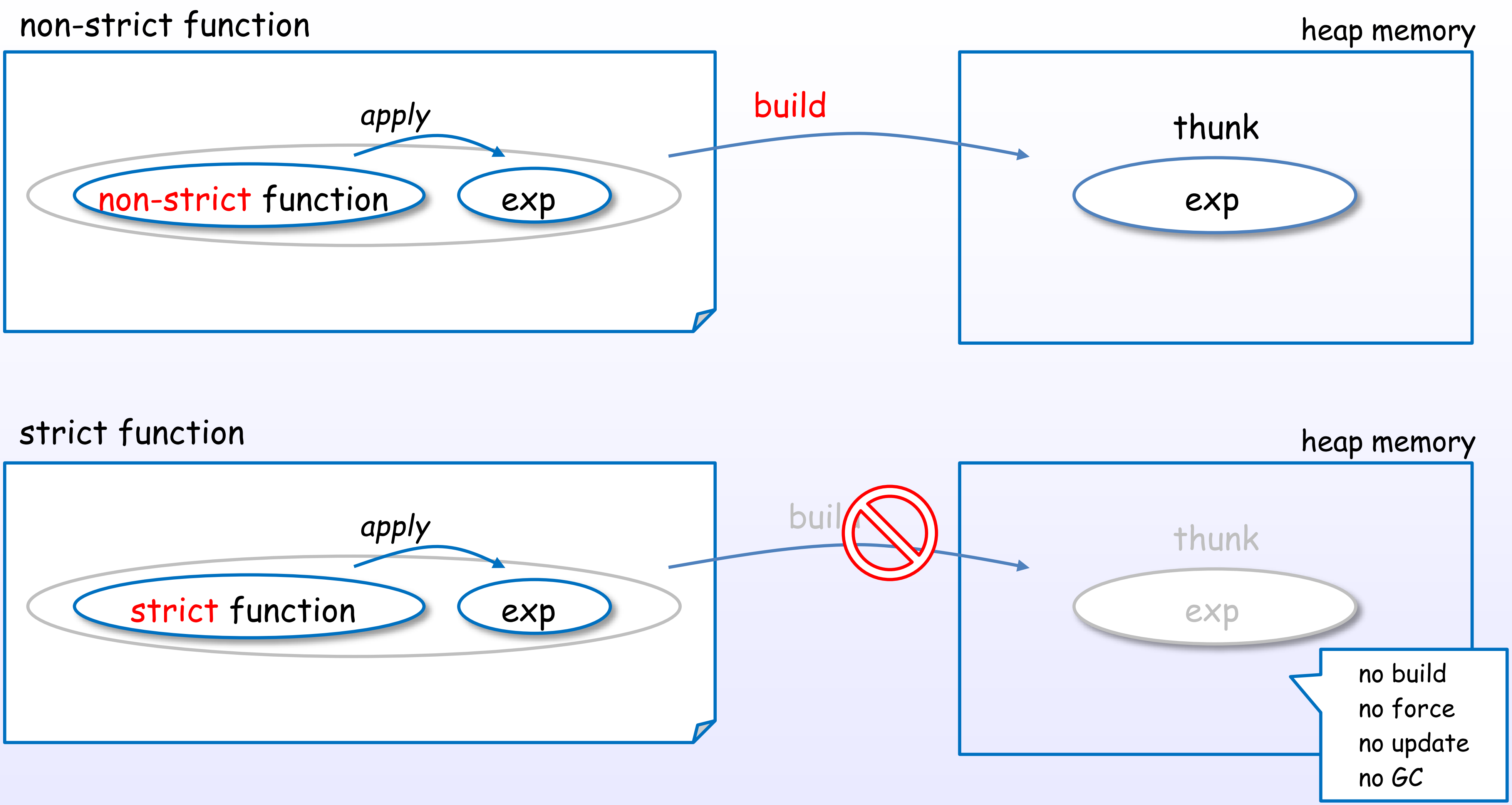
L -- second argument is "Lazy"

S -- first argument is "head-Strict"

GHC shows strictness analysis information with "-ddump-strsigs" and "-ddump-stranal".

References : [H15], [H16], [H2], [H4] Ch.22, [H8], [W6], [W3], [H13]

(1) Strictness analysis are used to avoid the thunk

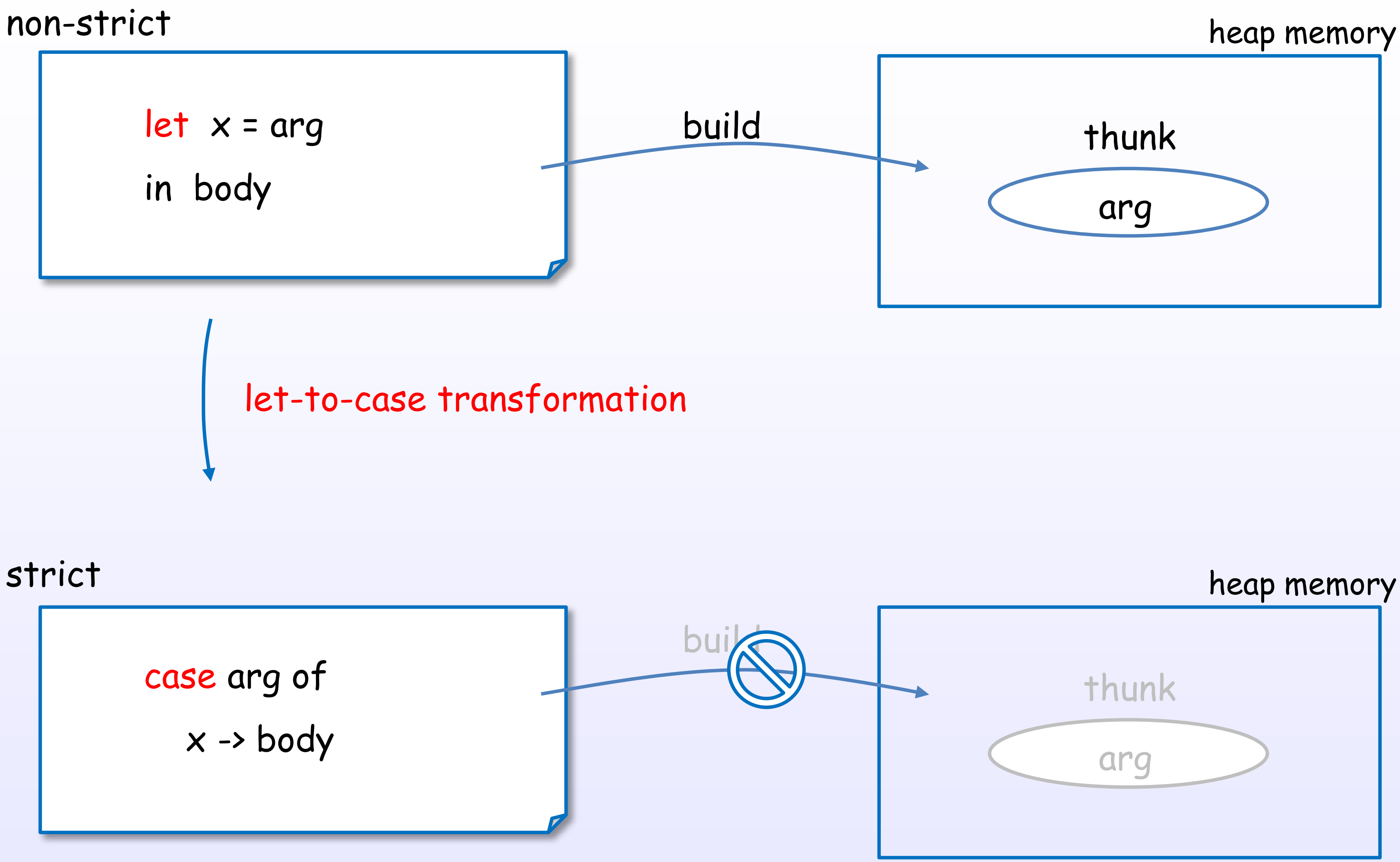


If GHC knows that a function is strict, arguments is evaluated before application.

GHC finds strict functions by "strictness analysis (demand analysis)".

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H17], [H13]

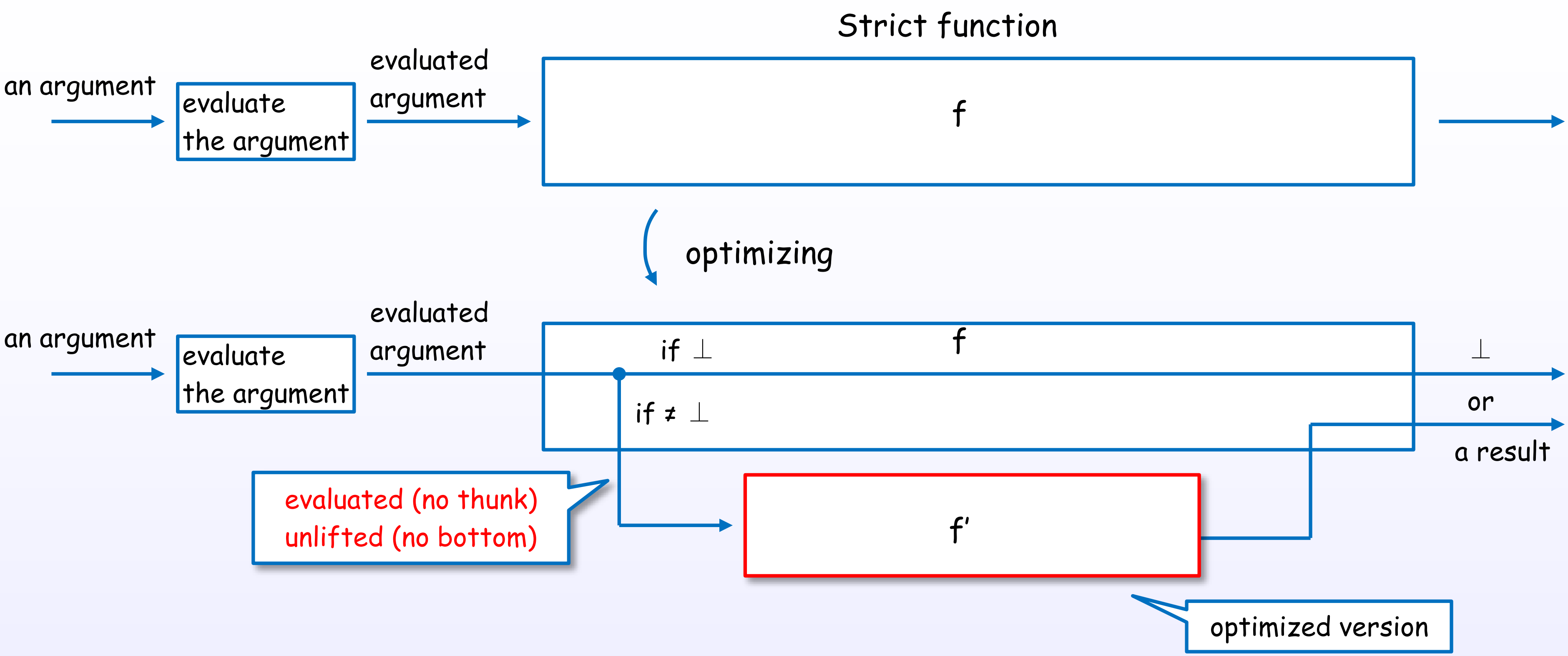
(1) Strictness analysis are used to avoid the thunk



If GHC knows that a function is strict, GHC performs let-to-case transformation.

References : [H8], [H4] Ch.22, [W6], [W3], [H15], [H17], [H13]

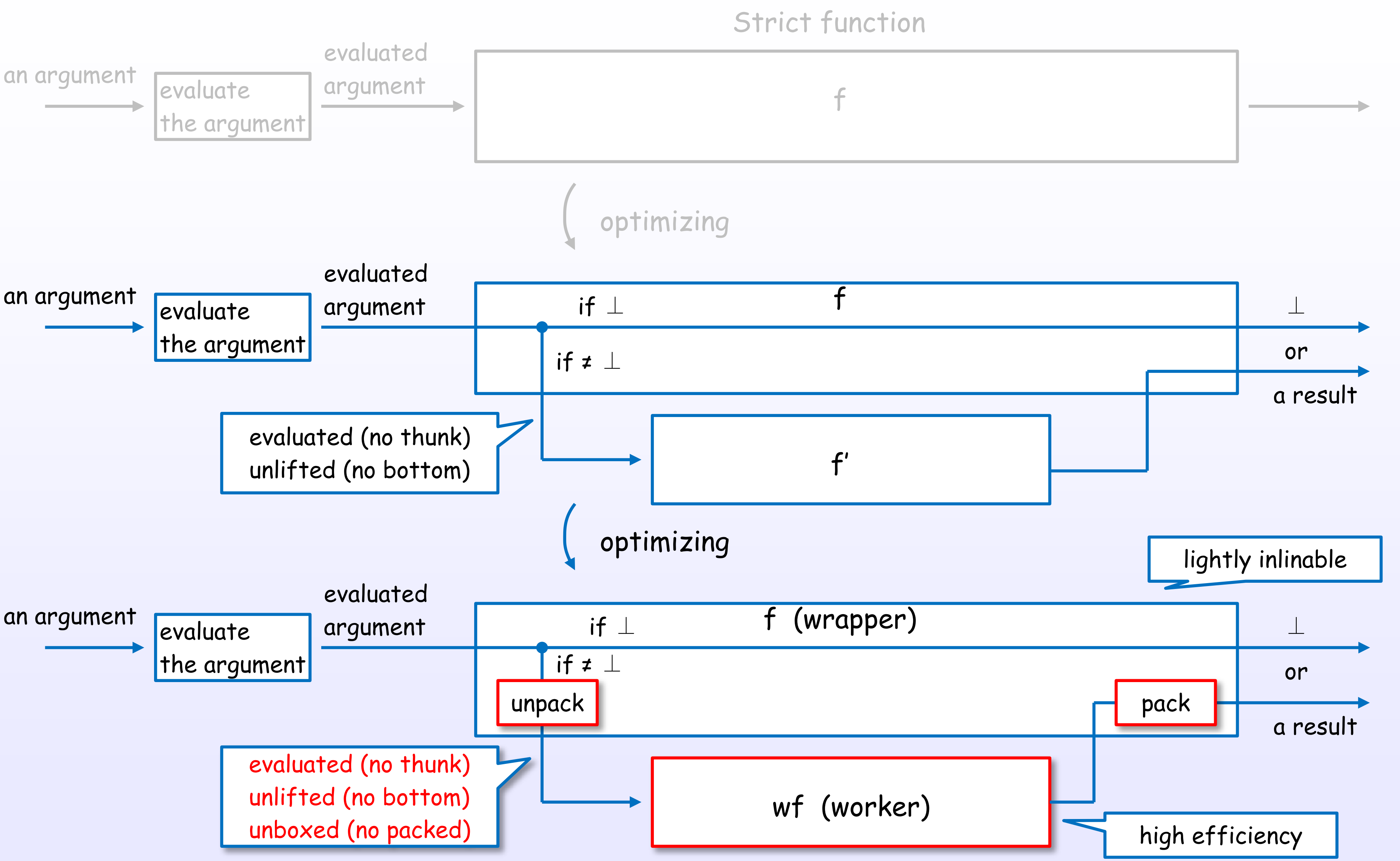
(2) Strictness analysis are also used to optimize



Strictness function can be optimized to assume no thunk, no bottom.

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H17], [H13]

(2) Strictness analysis are also used to optimize



Strictness function can be optimized to assume no thunk, no bottom, no packed.

References : [H4] Ch.22, [H8], [W6], [W3], [H15], [H17], [H13]

Sequential order

"seq" doesn't guarantee the evaluation order

specification

$$
\begin{aligned}
\text{seq } a \ b &= \perp, && \text{if } a = \perp \\
&= b, && \text{otherwise}
\end{aligned}
$$

strictness for each arguments

$$
\begin{aligned}
\text{seq } \perp \ b &= \perp && \text{// a is strict} \\
\text{seq } a \ \perp &= \perp && \text{// b is strict}
\end{aligned}
$$

"seq" function only guarantee that it is strict in both arguments.
This semantics property makes no operational guarantee about order of evaluation.

References : [H9], [D11], [H1] Ch.6, [S1]

"seq" and "pseq"

specification

$$
\begin{aligned}
\text{seq } a \; b &= \perp, && \text{if } a = \perp \\
&= b, && \text{otherwise}
\end{aligned}
$$

$\text{seq } \perp \; b = \perp$ // a is strict
$\text{seq } a \; \perp = \perp$ // b is strict

specification

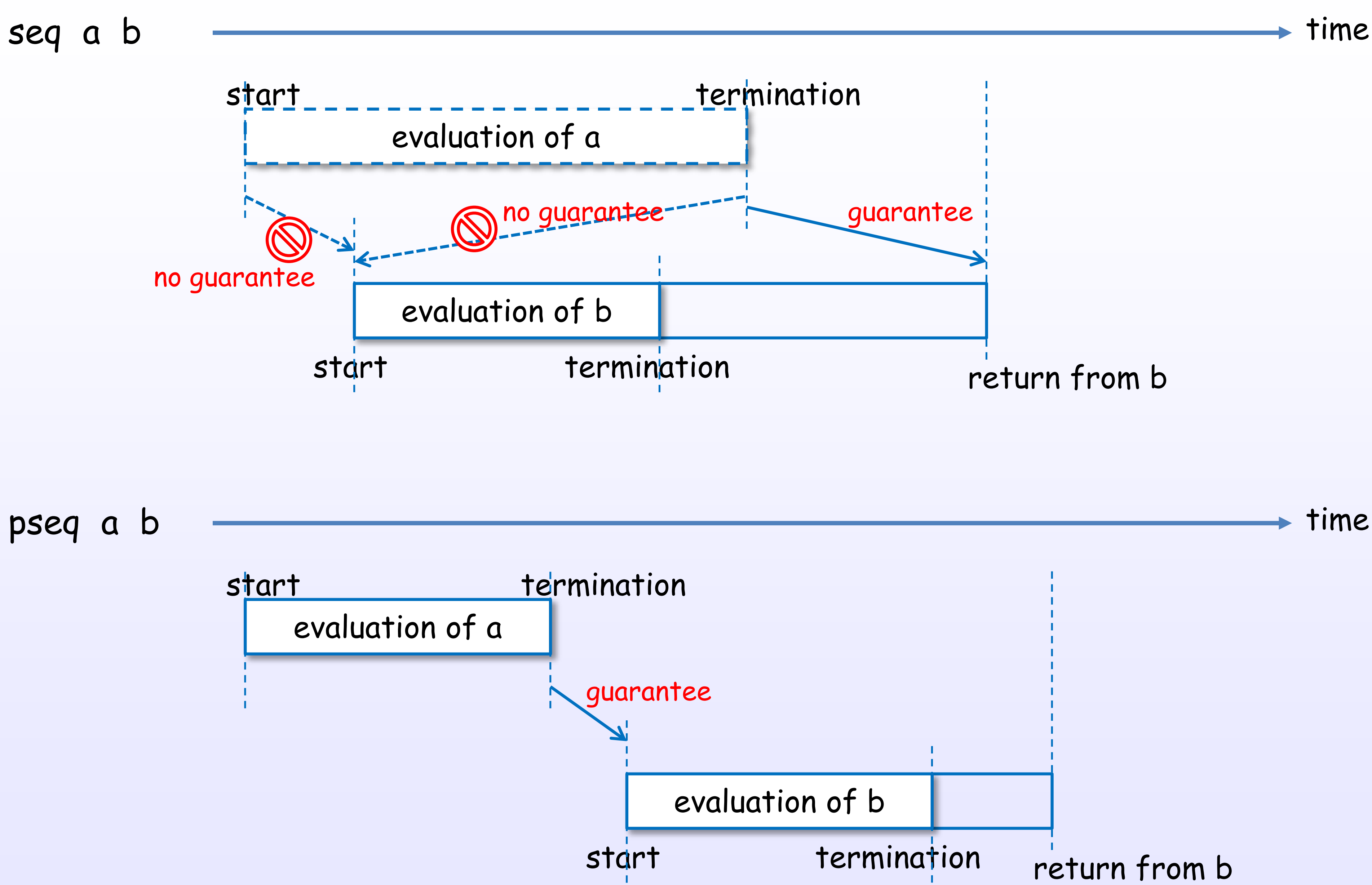
$$
\begin{aligned}
\text{pseq } a \; b &= \perp, && \text{if } a = \perp \\
&= b, && \text{otherwise}
\end{aligned}
$$

$\text{pseq } \perp \; b = \perp$ // a is strict
$\text{pseq } a \; \perp = \perp$ // b is strict

Both of denotational semantics are the same.
But "pseq" makes operational guarantee about order of evaluation.

References : [H9], [D11], [H1] Ch.6, [S1]

Evaluation order of “seq” and “pseq”



References : [H9], [D11], [H1] Ch.6, [S1]

Implementation of “seq” and “pseq”

specification

$$\textcolor{red}{seq}\ a\ b = \perp, \quad \text{if } a = \perp$$
$$= b, \quad \text{otherwise}$$

Haskell’s built-in

specification

$$\textcolor{red}{pseq}\ a\ b = \perp, \quad \text{if } a = \perp$$
$$= b, \quad \text{otherwise}$$

pseq x y = x `seq` lazy y

GHC’s “lazy” function restrains the strictness analysis.

“seq” is built-in function.
“pseq” is implemented by built-in functions (“seq” and “lazy”).

References : [H9], [D11], [H1] Ch.6, [H2] Ch.7, [S1]

7. Appendix

References

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[W6] Performance/Strictness
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Lazy,… zzz

to be as lazy as possible…